Stone Weierstrass Approx. Theorem

(Rudin, Principles of M.A)

Theorem If f is cts [=K] complex function on $[a,b]$ there exists a seq $\{P_n\}$ of polynomials, s.t.

$$\lim_{n\to\infty} P_n(x) = f(x) \quad (*)$$

uniformly on $[a,b]$.

If f is real func, then P_n can be taken to be real.

(*) Given $\varepsilon > 0$, $\exists\, n_0 \in \mathbb{N}$ st $|P_n(x) - f(x)| < \varepsilon$.

$\forall\, n \geq n_0$.

& $\forall\, x \in [a, b]$

Theorem S.W.   L03

Let $\mathcal{A}$ be an algebra (1)
of real continuous func.
on a compact set K.
If $\mathcal{A}$ separates points (2)
on K and if $\mathcal{A}$ vanishes
at no point of K (3)
then the uniform (4)
closure $\mathcal{B}$ of $\mathcal{A}$
is the set of all its real
valued fncts on K.

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Algebra: A family $\mathcal{A}$ of complex valued func. defined on a set E is said to be an algebra if (i) $f+g \in \mathcal{A}$

(ii) $fg \in \mathcal{A}$.

(iii) $cf \in \mathcal{A}$ $\forall c \in \mathbb{C}$.

(If we are considering only real functs, then in (iii) we replace $\mathbb{C}$ by $\mathbb{R}$)

Lecture 05/07.

$$s = \sum_{k=1}^{n} x_k \chi_{M_k}$$

$$\|s(t)\| = ? \quad \left(s : M \longrightarrow X \right)$$

$$s(t) = \sum_{k=1}^{n} x_k \chi_{M_k}(t)$$

$$= \begin{cases} x_k, & t \in M_k, \\ 0 & \text{altfel} \end{cases}$$

$$\|s(t)\| = \begin{cases} \|x_k\|, & t \in M_k \\ 0 & \text{altfel} \end{cases}$$