R-07
Convex and Nonsmooth Analysis
Viva Questions

Find the extreme points of the set

$$C = \left\{x \in \mathbb{R}^3 : \sqrt{x_1^2 + x_2^2} + |x_3| \leq 1\right\}.$$

Do you have knowledge about the norms can be represented as support functions?

Can you give an example of a closed unbounded convex set in $\mathbb{R}^2$ whose set of extreme set is bounded?

Can you give an example of a compact convex set in $\mathbb{R}^2$ whose set of extreme set is closed?

Can you give an example of a compact convex set in $\mathbb{R}^2$ whose set of extreme set is not closed?

Is the set of extreme points of a compact convex set with nonempty interior in $\mathbb{R}^3$ closed?

A non-empty compact convex sets are determined uniquely by their support functions. Is this statement true?

What is asymptotic cone of $C = \left\{x \in \mathbb{R}^3 : \sqrt{x_1^2 + x_2^2} + x_3 \leq 1\right\}$?

1. Can you give an example of a convex set whose face is not closed?
2. Can you give an example of a convex set whose face is not bounded?
3. If A is a face of F, F is a face of C, then A is a face of C. Now if A is a face of C, F is a face of C, $A \subseteq F$ then is A also a face of F? Justify.
4. Find faces and exposed faces of the set

$$C = \left\{x \in \mathbb{R}^3 : \sqrt{x_1^2 + x_2^2} + |x_3| \leq 1\right\}.$$

$$C = \{(x, y) \in \mathbb{R}^2 : x > 0\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0, y \geq 0\}.$$

$$C = \{x \in \mathbb{R}^3 : x_1^2 + x_2^2 \leq 1, x_3 = 0\}.$$

5. Find the polar of the cone at (0,0,0)

$$C = \{x \in \mathbb{R}^3 : |x_3| \leq x_2\}.$$

Find $\sigma_S(d) + \sigma_S(-d)$ for the set for $d = (1,1)$

$$S = \{x \in \mathbb{R}^2 : {x_1}^2 + 4{x_2}^2 = 4\}$$

Find the projection of (1,1,1) on

$$C = \left\{x \in \mathbb{R}^3 : \sqrt{x_2^2 + x_3^2} + |x_1| \leq 1\right\}.$$

Find the asymptotic cone of the set

$$S = \{x \in \mathbb{R}^2 : {x_1}^3 \leq x_2\}.$$

Hence find dom σ_S.

Give an example of a convex C and a supporting hyperplane $H_{s,r}$ such that

$C \cap H_{s,r}$ is singleton set say $\{x\}$ and $\text{cl}(C) \cap H_{s,r}$ is an unbounded set with x in its relative interior.

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Find the asymptotic cone of the set

$$S = \{x \in \mathbb{R}^2 : {x_1}^3 \leq x_2\}.$$

Hence find dom σ_S.

Give an example of a convex C and a supporting hyperplane $H_{s,r}$ such that $C \cap H_{s,r}$ is singleton set say $\{x\}$ and $\mathrm{cl}(C) \cap H_{s,r}$ is an unbounded set with x in its relative interior.

Theorem Let $S \subset \mathbb{R}^n$ and $C = \mathrm{co}S$. Any $x \in C \cap \mathrm{bd}(C)$ can be represented as a convex combination of at the most n elements of S.

Can you give an example of a set S in $\mathbb{R}^3$ such that $x \in C \cap \mathrm{bd}(C)$ can be represented as a convex combination of two elements of S, where $C = \mathrm{co}S$?

Find a supporting hyperplane to the set

$$C = \left\{x \in \mathbb{R}^3 : \sqrt{x_2^2 + x_3^2} + |x_1| \leq 1\right\}$$

at $(1,0,0)$ and $\left(-\frac{1}{2}, -\frac{1}{2}, 0\right)$.

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Is this a cone?

$$C = \{x \in \mathbb{R}^3 : |x_1| \leq x_3\}.$$

What is the polar of the cone?

If $f(x, y) = g(3x + 2y)$ and suppose $g'(7) = 5$, then find the directional derivative at the point $(1,2)$ and in the direction of $d = (2,3\sqrt{2})$.

Calculate the support function of the set

$$C = \text{co}\{(0,0), (1,1), (-1,1)\}.$$

What do you mean by inf convolution of two functions? What is the inf convolution of the real valued function defined on $\mathbb{R}$ as $f(x) = 1$ and $g(x) = -e^x$?

Is this a cone?

$$C = \{x \in \mathbb{R}^2 : |x_1| \leq x_2\}.$$

$$C = \{x \in \mathbb{R}^3 : |x_1| \leq x_3\}.$$

If yes, what is the polar of this cone?

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What are the tangent and normal cones of C_1, C_2 and $C_1 \cap C_2$ at origin?

$$C_1 = \{x \in \mathbb{R}^3 : x_1 \leq x_3\}$$
$$C_2 = \{x \in \mathbb{R}^3 : -x_1 \leq x_3\}.$$

What is the tangent cone of $A(C_1 \cap C_2)$ where $A: \mathbb{R}^3 \to \mathbb{R}^2$ defined as $A(x) = (x_1, x_3)$?

Is the function $f(x) = x^2 + |x|^3 + e^{|x|}$ a convex function?

What is the subdifferential of f at $x = 0$?

Separation, strict separation and proper separation

What conditions ensure this?

Is the convex hull of a closed set in $\mathbb{R}^n$ closed?

Let $S = \{(x, e^{-x}) : x \geq 0\} \cup \{(x, -e^{-x}) : x \geq 0\}$. Find $\mathrm{co}S$.

$$S = \left\{(x, y) : y \geq \frac{1}{1 + x^2}\right\}$$

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How do you define lower semicontinuous function?

Is this statement true?

A function f is lower semicontinuous at x if and only if for every sequence $\{x_n\}$ converging to x we have

$$f(x) \leq \liminf_{k\to\infty} f(x_k).$$

Let $C = \{(x_1, x_2) : {x_1}^2 + {x_2}^2 \leq 1\}$.Find a convex and a nonconvex set D such that

$$D - D = C.$$

Write an equation of a hyperplane in $\mathbb{R}^3$ orthogonal to the vector $(1,2,1)$ passing through $(1,-1,0)$.

Find the support function of the set $C = \left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\right\}$.

Every exposed point is an extreme point. Is the converse true? No.

$C = \{(x, y) : -1 \leq x \leq 1, -2 \leq y \leq 0\} \cup \{(x, y) : x^2 + y^2 \leq 1\}$

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Find the support function of the set $C = \left\{(x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\}$.

Find the support function of the set

$$C = \text{co}\{(0,0), (1,0), (0,1), (1,1)\}.$$

Find $D_C(x)$.

Find convex hull of the set

$$S = \{(x_1, x_2): {x_1}^2 + {x_2}^2 = 1, x_1 \ge 0\} \cup \{(2,0)\}.$$

Is

$$f(x) = \begin{cases} \max\{x^2 - 2,0, x\}, & -2 < x \le 4, \\ +\infty, & \text{otherwise}, \end{cases}$$

a closed convex function?

Give a bounded set C such that epigraphical hull is epi$f$.

Find the subdifferential of the function

$$f(x) = \max\{x^2 - 2,0, x\}$$

at points of nondifferentiability.

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Find convex hull of the set

$$S = \{(x, \sqrt{x}) : x \geq 0\}.$$

Find the subdifferential of the function

$$f(x) = \max\{x^2 - 2, 0, x\}$$

at points of nondifferentiability.

Can you establish mean value theorem for the points $x = 0$ and $y = 3$?

Find the infimal convolution of the functions

$$f(x) = x^2 - 1,\ g(x) = 2x.$$

Give an example of a subadditive function which is not positively homogeneous.

Give an example of a positively homogeneous function which is not subadditive.

Find convex hull of the set

$$S = \{(x, \sqrt{x}) : x \geq 0\} \cup \{(x, \sqrt{-x}) : x < 0\}.$$

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Find the distance function of the set

$$C = \text{co}\{(0,0), (1,0), (0,1)\}.$$

Is the function $f(x) = x^2 + |x|^3 + e^{|x|}$ a convex function?

Is this a strongly convex function?

Is this a cone?

$$C = \{x \in \mathbb{R}^3 : |x_1| \leq x_3\}.$$

What is the polar of the cone?