Theory of similarity for heat transfer

For compressible fluid (ideal gas)

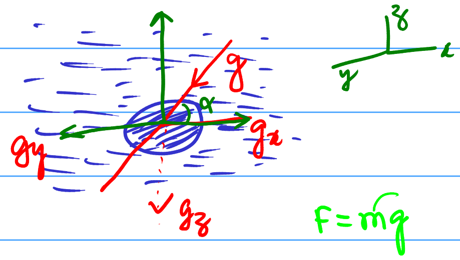
$$\rho = \rho(p, T) \qquad (\text{for perfect} \quad p = \rho R T)$$

Let us consider buoyancy force as external body force, then the

Let $\vec{F}$ be body force per unit volume

$$\vec{F} = X\hat{i} + Y\hat{j} + Z\hat{k} = \rho\vec{g} = \rho(g_x\hat{i} + g_y\hat{j} + g_z\hat{k})$$

$$X = \rho g_x, \quad Y = \rho g_y, \quad Z = \rho g_z$$



$F = mg$

Let, the reference state of the fluid is given by $\rho_\infty, p_\infty, T_\infty$

$$\rho = \rho_\infty + \left(\frac{\partial \rho}{\partial T}\right)_{p, T_\infty}(T - T_\infty) + \left(\frac{\partial \rho}{\partial p}\right)_{T, p = p_\infty}(p - p_\infty),$$

speed of small disturbance into the fluid

Thermal expansion coefficient.

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{p, T = T_\infty} =$$

Let V is specific volume

$$V = \frac{1}{\rho}\Bigg|_{\rho = \rho_\infty,\, p = p_\infty,\, T = T_\infty}$$

$$= \rho_\infty\left(\frac{\partial}{\partial T}\left(+\frac{1}{\rho}\right)\right)_{p, T = T_\infty}$$

$$= \rho_\infty \times -\frac{1}{\rho_\infty^2}\left(\frac{\partial \rho}{\partial T}\right)_{p, T = T_\infty}$$

$$\Rightarrow \quad -\rho_\infty\beta = \left(\frac{\partial \rho}{\partial T}\right)_{p, T = T_\infty},$$

$$c_\infty^2 = \left(\frac{\partial p}{\partial \rho}\right)_{T, p = p_\infty},$$

$p = k\rho^\gamma$ (isentropic eq^n)

$$\gamma = \frac{c_p}{c_v}$$

$$\left(\frac{\partial p}{\partial \rho}\right)_{T, p = p_\infty} = \frac{\gamma p}{\rho} = c_\infty^2$$

Isothermal speed of sound

$$\left(\frac{\partial p}{\partial \rho}\right)_T = c_\infty^2$$

$p = \rho R T$

$$\left(\frac{\partial p}{\partial \rho}\right)_{T, p_\infty} = RT = \frac{p}{\rho} = \frac{\gamma p}{\rho} \times \frac{1}{\gamma} = \frac{c_\infty^2}{\gamma}$$

$\theta = T - T_\infty$

$$\rho = \rho_\infty - \rho_\infty\beta(T - T_\infty) + \frac{\gamma}{c_\infty^2}(p - p_\infty)$$

If density do not vary much on pressure, then $\rho = \rho_\infty - \rho_\infty\beta(T - T_\infty)$

$$X = \rho g_x = \rho_\infty g_x - \rho_\infty\beta g_x(T - T_\infty)$$

$$\frac{\partial p_{st}}{\partial x}i + \frac{\partial p_{st}}{\partial y}j + \frac{\partial p_{st}}{\partial z}k$$

Let us consider static pressure $\nabla p_{ST} = \rho_0 \vec{g} = \rho_\infty (g_x i + g_y j + g_z k)$,

and substract from the NS eqn of motion. Then the governing eqn of motion for steady flow of compressible fluid will be

$p = p + p_{st}$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0, \quad \vec{w} = ui + vj + wk \qquad \text{— (1)}$$

$$\rho\frac{du}{dt} = \rho\left(\underbrace{\frac{\partial u}{\partial t}}_{=0} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = X - \frac{\partial P}{\partial x} + \mu\left[\nabla^2 u + \frac{1}{3}\frac{\partial}{\partial x}(\nabla\cdot\vec{w})\right]$$

where $P = p + p_{st}$

$$\rho\frac{d\vec{w}}{dt} = \rho\vec{F} - \nabla p + \mu\nabla^2\vec{w} + \frac{1}{3}\mu\nabla(\nabla\cdot\vec{w})$$ ✓ (Chorlton's Book) page 324.

$$\rho\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = \cancel{\rho_\infty g_x} - \rho_\infty g_x \beta\theta - \frac{\partial p}{\partial x} - \underbrace{\cancel{\frac{\partial p_{st}}{\partial x}}}_{= \rho_\infty g_x} + \mu\left[\nabla^2 u + \frac{1}{3}\frac{\partial}{\partial x}(\nabla\cdot\vec{w})\right]$$

$$\rho\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} + \rho w\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial x} - \rho_\infty g_x\beta\theta + \mu\left[\nabla^2 u + \frac{1}{3}\frac{\partial}{\partial x}(\nabla\cdot\vec{w})\right] \qquad \text{— (2)}$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] = \qquad \text{— (3)}$$

$$\rho\left[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] = \qquad \text{— (4)}$$

$\frac{dp}{dt} = \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \cdots$ For constant thermal conductivity, the energy equation will be

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = \underbrace{u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}}_{=0 \text{ for incompressible fluid}} + K\nabla^2 T + \mu\Phi \qquad (5)$$

If compressible fluid is perfect gas then $p = \rho RT$ (6).

variables to be find out $\underline{u, v, w, \rho, p, T}$ and the system of eqns to be used are (1) – (7)

For incompressible fluid, ρ = constant, $\operatorname{div}\vec{w} = 0$

Now, we non-dimensionalize the eq^n ① - ⑦ and find out non-dimensional group of numbers.

Let, l be char. length, ρ_∞, U_∞ and $\rho_\infty U_\infty^2$ as char. density, velocity and char. pressure then

✓ $x = l x^*$, $y = y^* l$, $z = z^* l$, $u = u^* U_\infty$, $v = v^* U_\infty$, $\omega = \omega^* U_\infty$,

$p = p^* \rho_\infty U_\infty^2$,

$\theta = T - T_\infty = \theta^* (\Delta T)_0$, $\quad \Delta T_0 = T_w - T_\infty$ ← temp at large distance from the wall

T_w → wall temp

Use these transformation to derive the non-dim equations of motion. and find out non-dim. numbers (independent to each).

Assignment

Do the page ~~272 and~~ 272 to 276 as assignment.