Your Roll Number: ....P2109.....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI

M.Phil/Ph.D. Coursework Examinations, July 2022

MATH21-R08: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper. • Question no. 1 is compulsory. • Answer ANY 4 questions among other six questions. • All the symbols have their usual meaning.

(1) (a) Define a shock wave for a one-dimensional conservation law and derive the Lax entropy criterion. [1+2 Marks]

(b) Write the conservation and matrix form of equations for one-dimensional flow of dusty gas. [2 Marks]

(c) Derive the Rankine-Hugoniot jump condition for ideal relaxing gas [2 Marks]

(d) Enumerate the differences between the natural and forced convection. [2 Marks]

(e) Write the system of equations for two dimensional steady thermal boundary layer flow of compressible fluid. [3 Marks]

(f) Describe the Reynolds number and adiabatic wall. [2 Marks]

(2) (a) Define the self-similar weak solution and Lie group of transformation. State and prove first fundamental theorem of Lie. [2+5 Marks]

(b) Define the rarefaction wave with a suitable example. Find the eigenvalues and eigenvectors of linearalized gas dynamics equations. [2+5 Marks]

(3) (a) Define the infinitesimal generators. Determine the solution of linearalized gas dynamics equations from decoupled form. [2+5 Marks]

(b) What do you understand by Riemann problem for non-convex flux function and non-classical shocks. Solve the Burgers inviscid equation [2+5 Marks]

with following initial data $u(x,0) = \begin{cases} 1, & x < 0 \\ -1, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$

(4) (a) Solve the following equation: $u_t + (u^2/2)_x = 0, x \in R, t > 0$ [7 Marks]

with initial data $u(x,0) = \begin{cases} 1, & -\infty < x \leq -1 \\ 0, & -1 < x \leq 0 \\ 2, & 0 < x \leq 1 \\ 0, & x > 1 \end{cases}$

(b) Give two real life applications of conservation law. Solve the signaling problem $u_t + u_x = 0, x > 0, t \in R$, with initial data $u(x, 0) = u_0($[illegible] [2+5 Marks]

(5) (a) Define the similar solution and similarity requirement. Discuss the types of potential flow for the existence of similar solution of boundary layer equations for plane steady flow and obtain the Falkner Scan's equation. [2+7 Marks]

(b) Compare the mass and momentum thickness of a velocity boundary layer flow over a flat plate. [5 Marks]

(6) (a) Perform the non-dimensional and order analysis on equations of motion of two dimensional viscous incompressible fluid over a slender body and derive the equations for the boundary layer flow. [7 Marks]

(b) Derive the energy equation for the perfect gas flow. Write the corresponding equations for the incompressible fluid. [7 Marks]

(7) (a) Compute the skin friction on both side of a wetted flat plate of the length l and the width b due to a boundary layer flow of viscosity μ, density ρ and uniform speed U_∞. [5 Marks]

(b) Write the basic equations that describe the thermal boundary layers in a parallel forced flow past a flat plate at zero incidence. Neglecting frictional heat, compute the distribution of temperature, heat flux from plate to fluid and rate of heat transfer from both sides of plate(length l and width b). [2+7 Marks]