MATH21-R07: CONVEX AND NONSMOOTH ANALYSIS

Attempt any five questions.

1. Find both co$S$ and $\overline{\text{co}}S$ for a set S in $\mathbb{R}^2$ both in the set form and geometrically where
$$S = \{(x, e^{-x}) : x \geq 0\} \cup \{(x, -e^{-x}) : x > 0\}.$$
2. Find faces and exposed faces of the set
$$C = \{x \in \mathbb{R}^3 : 2x_1^2 + x_2^2 + 3x_3^2 \leq 1, x_3 \geq 0\}.$$
3. Find the projection of (1,1,1) on the set
$$C = \{x \in \mathbb{R}^3 : x_1^2 + x_2^2 + x_3^2 \leq 1\}$$
and find supporting hyerplane passing through the point of projection.
4. Give an example of a convex C in $\mathbb{R}^2$, a point $x \in \mathbb{R}^2$ and a supporting hyperplane $H_{s,r}$ such that $C \cap H_{s,r} = \{x\}$, and $\mathrm{cl}(C) \cap H_{s,r}$ is an unbounded set with x in its relative interior.
5. Find the asymptotic cone of the set
$$S = \{x \in \mathbb{R}^2 : x_1^3 \leq x_2\}.$$
Also find another nonconvex set with same asymptotic cone.
6. Find the tangent and normal cones of $C_1 \cap C_2$ at origin where
$$C_1 = \{x \in \mathbb{R}^3 : x_1 \leq x_3\}$$
$$C_2 = \{x \in \mathbb{R}^3 : -x_1 \leq x_3\}.$$
7. For the sets C_1 and C_2 considered in Q. 6, find the tangent cone of $A(C_1 \cap C_2)$ where $A : \mathbb{R}^3 \to \mathbb{R}^2$ is defined as $A(x) = (x_1, x_3)$?
8. Let $C = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\}$. Find a nonconvex set D such that $D - D = C$.

$\delta = (1,1,1) - \frac{1}{\sqrt{3}}(1,1,1)$

$\exists x \in ri\ cl(C) \cap H_{s,r})$

$\delta = \frac{\sqrt{3}-1}{\sqrt{3}}(1,1,1)$

$r = \sqrt{3}-1$

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Attempt all questions

6. Find the subdifferential of the function f at each $x = 0$ where
   (c) $f(x) = \max\{3x, x^2, -2x\}$,
   (d) $f(x) = x^2 + |x|^3 + e^{|x|}$.
7. Calculate the support function of the set
$$C = \mathrm{co}\{(0,0), (1,1), (-1,1)\}.$$
8. Find the inf convolution of the real valued functions f and g defined on $\mathbb{R}$ as
   (c) $f(x) = 1$ and $g(x) = -e^x$,
   (d) $f(x) = 2x^4$ and $g(x) = x$.
9. If $f(x, y) = g(x + 2y)$ and $g(x) = x^2 - x$, where $x, y \in \mathbb{R}$, then find the directional derivative of f at the point (1,2) in the direction of $d = (2, -3\sqrt{2})$. Is the function f differentiable? Justify.
10. Give an example of a subadditive function which is not positively homogeneous. Also, give an example of a positively homogeneous function which is not subadditive.

$1+2t+4-6\sqrt{2}t$

$5+(2-6\sqrt{2})t$