Your Roll Number: ....

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF DELHI
M.Phil and Ph.D. Coursework Examinations, August 2021

MATH20-R08: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Time: 3 hours | Maximum Marks: 70

Instructions: • Write your roll number on the space provided at the top of this page immediately on receipt of this question paper. • Question no. **1** is compulsory. Attempt **5** questions in all. • Answer ANY **2** questions from each section B and C • All the symbols have their usual meaning.

Section A

(Answer all the parts)

(1) (a) Derive the relation for temperature increase through adiabatic compression. [3 Marks]

(b) Compute the ratio of inertial force and viscous force per unit volume for steady viscous flow of an incompressible fluid. Explain the corresponding non-dimensional number. [4 Marks]

(c) Explain the constitutive relations with examples. [3 Marks]

(d) Describe the Nusselt number. [2 Marks]

(e) Explain the different mode of heat transfer with necessary equations. [2 Marks]

Section B

(Answer any TWO questions)

(2) (a) Define hyperbolic system of conservation laws. Describe the method of finding the solution of conservation laws $U_t + AU_x = 0, U(x,0) = U_0(x)$, where A is constant matrix with m distinct eigenvalues. [7 Marks]

(b) Derive mass and momentum equation for one dimensional fluid flow in planer, cylindrical and spherical symmetry. [7 Marks]

(3) (a) Derive the jump condition across the discontinuity in solution of a Riemann problem. Is single conservation equation can have two jump conditions? Justify your answer. [7 Marks]

(b) Discuss the uniqueness of the solution of an IVP [7 Marks]
$u_t + uu_x = 0, x \in \mathbb{R}, t > 0$

$$u(x,0) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } 0 < x, \end{cases}$$

(4) (a) Derive the system of linearised equations of gas dynamics. Check [8 Marks]

its hyperbolicity. And hence derive the wave equations for density and speed of the gas flow.

(b) Derive the relation for weak solution of an IVP $u_t + (\phi(u))_x = 0, x \in \mathbb{R}, t > 0$,under IC $u(x, 0) = u_0(x)$. Differentiate between genuine and weak solution of the IVP. [6 Marks]

Section C

(Answer any TWO questions)

(5) (a) Define mass and momentum thickness of boundary layer and derive their expression over a long flat plate. [6 Marks]

(b) Derive the Blasius's equation with conditions for boundary layer flow over a semi-infinite flat plate. Find the expression for shear stress at the wall and drag coefficient for the flow. [8 Marks]

(6) (a) Derive the energy equation for motion of compressible fluid under heat conduction. Derive the corresponding equation for constant thermal conductivity of perfect gas and incompressible fluid. [7+3 Marks]

(b) Describe the difference between forced and natural convection, Grashof Number, and Prandltl Number. . [4 Marks]

(7) (a) Describe the simplification of the basic equations of thermal boundary layer flow. Write the corresponding equation for two dimensional compressible fluid flow. [7 Marks]

(b) Write the system of basic equations and boundary condition for the thermal boundary layer in a natural convection over a vertical hot plate. Using similarity transformation reduce the equations in system of ODEs. Compute the total heat transfer from length l and breadth b of the plate. Also compute the mean Nusselt number over the plate. [7 Marks]

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