IN-HOUSE EXAMINATION July 2021

M.Phil/Ph.D. Course Work Exam

Department of Mathematics, University of Delhi

MATH20-R08,Hyperbolic system of conservation laws and boundary layer theory

Time : 1 hrs and 30 minutes Max Marks : 20

Note: Answer **ALL** questions from **Section A** and any **three** questions from **Section B**. Write your Name, Roll Number, Course name of the paper on the first page and page No. on each subsequent page of your answer script.

Section A

In each of Questions 1-5, write the correct options out of the four choices (A), (B), (C) and (D). There may be **more than one options true**. Marks will be only awarded when you answer correct option(s).

(1) Consider the following statements:

(I) The differential form of conservation laws is valid in a domain containing discontinuity.

(II) Integral form of conservation law is more general than the differential form.

(III) Fourier's law of heat conduction is a conservation law.

Which of the following options **is/are correct**? [1]

(A) (I), (II) and (III) are true.

(B) (III)is true but (I) and (II) are false.

(C) (II) is true but (I) and (III) are false.

(D) (I) is true but (II) and (III) are false.

(2) Which of the following options **is/are correct**? [1]

(A) Inviscid Burger equation deal with a reaction diffusion process.

(B) The equations of continuity, motion and energy do not form a complete set of equations for description of compressible fluid motion.

(C) The shock conditions are not consistent with the notion of weak solution of a conservation laws.

(D) The entropy condition ensures the existence of a unique solution of a conservation laws.

(3) Consider the following statements:

(I) Order analysis reduces the Navier Stokes equation of motion into Prandtl's boundary layer equations.

(II) Magnitude of velocity gradient normal to the wall separate the lightly viscous flow over the wall into outer flow and boundary layer flow.

(III) For high Reynolds number flow ($Re >> 1$), the inertial forces are dominant over viscous forces near the interfaces and boundary layers.

Which of the following options **is/are correct**? [1]

(A) (I) and (II) are true and (III) is false.

(B) (I),(II)and (III) are false.

(C) (I),(II) and (III) are true.

(D) (I)and (II) are false and (III) is true.

(4) Consider the following statements:
(I) - The non-dimensional boundary layer thickness tends to zero for high Reynolds number flow.
(II) - The boundary layer thickness is inversely proportional to the Reynolds number.

Which one of the following options is correct? [1]
(A) Statement (I) and (II) both are true and Statement (II) is correct explanation of statement (I).
(B) Statement (I) and (II) both are true but Statement (II) is not a correct explanation of statement (I).
(C) Statement (I) is true but (II) is false.
(D) Statement (II) is true but (I) is false

(5) Which of the following options **is/are correct**? [1]
(A) For convex flux function the shock speed is intermediate to the characteristic speed on both side of shock.
(B) Riemann's problem is solved uniquely either by shock wave or rarefaction wave depending on the initial condition and the flux function.
(C) Linearized equations of gas dynamics are strict hyperbolic conservation laws throughout the flow field.
(D) A weak solution of a conservation laws must be smooth.

Section B

(6) Describe the method of finding the solution of Riemann problem for strict hyperbolic, constant coefficient PDEs $U_t + AU_x = 0, -\infty < x < \infty, t > 0$,under IC $U(x,0) = U_L, x < 0; U(x,0) = U_R, x > 0$. [5]

(7) Define similarity solution and similarity equation. Find the similarity transformation for the heat equation $u_t = ku_{xx}, u(x,0) = 0, x > 0, u(x,t) \to 0, x \to \infty, u(0,t) = u_0, t > 0$ and using the transformation reduce the problem into ordinary differential equation. [5]

(8) Find the (i) continuous solution for $t < 1$ and (ii) using shock fitting find the solution for $t > 1$ of an IVP
$u_t + uu_x = 0, x \in \mathbb{R}, t > 0$

$$u(x,0) = \begin{cases} 1, & \text{if } x < 0 \\ 1 - x, & \text{if } 0 < x < 1 \\ 0, & \text{if } x > 1, \end{cases}$$

[5]

(9) Write the basic equations of motion and boundary conditions for two dimensional viscous incompressible fluid over a slender body, and perform the non-dimensional and order analysis to derive the equations for the laminar velocity boundary layer. [5]

....