MATH20-R06: TOPOLOGY AND MIXING

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Topological Transitivity: Examples and properties, Topological mixing: Examples and Properties, Transitivity and limit sets for continuous interval maps, Characterizing topological mixing in terms of topological transitivity for continuous interval maps, Sensitive dependence on initial conditions, Devaney's definition of chaos, Logistic maps and shift maps as chaotic maps, Period three implies chaos, Relation between transitivity and chaos on intervals**,** Various other definitions of chaos and their interrelationships.

Topological Entropy: Definition using open covers, Examples and properties, Bowen's definition of topological entropy, Equivalence of two definitions, Topological version of Kolmogorov–Sinai theorem, Topological entropy of an expansive homeomorphism, of the two sided shift, of the topological Markov chain, of any homeomorphism of the unit circle, of any homeomorphism of closed unit interval, an upper bound for the topological entropy of a diffeomorphism of a finite dimensional Riemannian manifold.

References

1. **Lluís Alsedà, Jaume Llibre & Michał Misiurewicz**, *Combinatorial Dynamics and Entropy in Dimension One*, Advanced Series in Nonlinear Dynamics, World Scientific, 2000.
2. **Louis S. Block & William A. Coppel**, *Dynamics in One Dimension*, Springer, 2014.
3. **Michael Brin & Garrett Stuck**, *Introduction to Dynamical Systems*, Cambridge University Press, 2015.
4. **Robert L. Devaney**, *A First Course in Chaotic Dynamical Systems*, CRC Press, 2018.
5. **Clark Robinson**, *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, CRC press, 1998.
6. **S. Ruette**, *Chaos for Continuous Interval Maps: A Survey of Relationship Between Various Kinds of Chaos*, 2018.
7. **Peter Walters**, *An Introduction to Ergodic Theory*, Springer, 2000.

MATH20-R07: CONVEX AND NONSMOOTH ANALYSIS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Convex sets, Convexity-preserving operations for a set, Relative interior, Asymptotic cone, Extreme points, Face, Projection operator, Separation theorems, Bouligand tangent and normal cones.

Convex functions, Closedness, Affinity, Epigraphical hull and lower-bound function of a set, Functional operations preserving convexity of function, Infimal convolution, Convex hull and closed convex hull of a function, Continuity properties; Sublinear functions, Support function, Calculus of support functions, Norms and their duals, Polarity.

Subdifferential of convex functions, Geometric construction, interpretation and properties of subdifferentials, Minimality conditions, Mean-value theorem; Calculus rule with subdifferentials.

References

1. **Jonathan M. Borwein & Adrian S. Lewis**, *Convex Analysis and Nonlinear Optimization: Theory and Examples*, CMS Books in Mathematics, Springer, 2006.
2. **Jean-Baptiste Hiriart-Urruty & Claude Lemaréchal**, *Fundamentals of Convex Analysis*, Springer, 2004.
3. **Boris S. Mordukhovich & Nguyen Mau Nam**, *An Easy Path to Convex Analysis and Applications*, Morgan & Claypool, 2014.
4. **R. Tyrrell Rockafellar**, *Convex Analysis*, Princeton University Press, 1997.
5. **C. Zălinenscu,** *Convex Analysis in General Vector Spaces,* World Scientific, 2002.

MATH20-R08: HYPERBOLIC SYSTEM OF CONSERVATION LAWS AND BOUNDARY LAYER THEORY

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Hyperbolic system of conservation laws: Fundamental concepts and examples, Scalar and system of conservation laws, Riemann Problem, Entropy condition, Classical and non-classical shocks, Similarity method.
Boundary layer theory: Laminar boundary layer, Turbulent flow, Turbulent boundary layer; Heat and Mass transfer, conduction, convection and radiation; Thermal boundary layer; Modeling and method of solution of the problems.

References

1. **G. B. Whitham**, *Linear and Nonlinear Waves*, John Wiley, 1999.
2. **Vishnu D. Sharma**, *Quasilinear Hyperbolic Systems, Compressible Flows and Waves*, CRC, 2010.
3. **Philippe G. LeFloch**, *Hyperbolic Systems of Conservation Laws: The Theory of Classical and Nonclassical Shock Waves*, Springer Basel AG, 2002.
4. **Hermann Schlichting & Klaus Gersten**, *Boundary-Layer Theory*, Springer, 2017.
5. **Tuncer Cebeci**, *Analysis of Turbulent Flows*, Elsevier, 2004.
6. **J.P. Holman & Souvik Bhattacharyya**, *Heat Transfer in SI Units*, Tata McGraw-Hill, 2011.
7. **George W. Bluman & Sukeyuki Kumei**, *Symmetries and Differential Equations*, Springer, New York, 1996.
8. **Eleuterio F. Toro**, *Riemann Solvers and Numerical Methods for Fluid Dynamics*: *A Practical Introduction*, Springer, 2009.

MATH20-R09: PARTIAL DIFFERENTIAL EQUATIONS: THEORY AND NUMERICS

Total Marks: **100 (Theory: 70, Internal Assessment: 30)**
Duration of Examination: **3 Hrs.**
Workload: **4 Lectures per week.** Credits: **4**

Maximum principles for second order linear parabolic, elliptic and hyperbolic partial differential equations; Weak solutions for second order linear parabolic, elliptic and hyperbolic partial differential equations; Lax–Milgram theorem, Local existence, uniqueness and regularity results for second order linear parabolic, elliptic and hyperbolic partial differential equations.