3.5 FORMULATING AND SOLVING LINEAR PROGRAMMING MODELS ON A SPREADSHEET

Spreadsheet software, such as Excel and its Solver, is a popular tool for analyzing and solving small linear programming problems. The main features of a linear programming model, including all its parameters, can be easily entered onto a spreadsheet. However, spreadsheet software can do much more than just display data. If we include some additional information, the spreadsheet can be used to quickly analyze potential solutions. For example, a potential solution can be checked to see if it is feasible and what Z value (profit or cost) it achieves. Much of the power of the spreadsheet lies in its ability to immediately reveal the results of any changes made in the solution.

In addition, Solver can quickly apply the simplex method to find an optimal solution for the model. We will describe how this is done in the latter part of this section.

To illustrate this process of formulating and solving linear programming models on a spreadsheet, we now return to the Wyndor example introduced in Sec. 3.1.

Formulating the Model on a Spreadsheet

Figure 3.14 displays the Wyndor problem by transferring the data from Table 3.1 onto a spreadsheet. (Columns E and F are being reserved for later entries described below.) We will refer to the cells showing the data as **data cells.** These cells are lightly shaded to distinguish them from other cells in the spreadsheet.[8]

FIGURE 3.14
The initial spreadsheet for the Wyndor problem after transferring the data from Table 3.1 into data cells.

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Profit Per Batch ($000)	3	5			
5							Hours
6			Hours Used Per Batch Produced				Available
7		Plant 1	1	0			4
8		Plant 2	0	2			12
9		Plant 3	3	2			18

[8]Borders and cell shading can be added by using the borders menu button and the fill color menu button on the Home tab.

An Application Vignette

Welch's, Inc., is the world's largest processor of Concord and Niagara grapes, with net sales of $650 million in 2012. Such products as Welch's grape jelly and Welch's grape juice have been enjoyed by generations of American consumers.

Every September, growers begin delivering grapes to processing plants that then press the raw grapes into juice. Time must pass before the grape juice is ready for conversion into finished jams, jellies, juices, and concentrates.

Deciding how to use the grape crop is a complex task given changing demand and uncertain crop quality and quantity. Typical decisions include what recipes to use for major product groups, the transfer of grape juice between plants, and the mode of transportation for these transfers.

Because Welch's lacked a formal system for optimizing raw material movement and the recipes used for production, an OR team developed a preliminary linear programming model. This was a large model with 8,000 decision variables that focused on the component level of detail. Small-scale testing proved that the model worked.

To make the model more useful, the team then revised it by aggregating demand by product group rather than by component. This reduced its size to 324 decision variables and 361 functional constraints. *The model then was incorporated into a spreadsheet.*

The company has run the continually updated version of this *spreadsheet model* each month since 1994 to provide senior management with information on the optimal logistics plan generated by the Solver. *The savings* from using and optimizing this model *were approximately* **$150,000** *in the first year alone.* A major advantage of incorporating the linear programming model into a spreadsheet has been the ease of explaining the model to managers with differing levels of mathematical understanding. This has led to a widespread appreciation of the operations research approach for both this application and others.

Source: E. W. Schuster and S. J. Allen, "Raw Material Management at Welch's, Inc.," *Interfaces,* **28**(5): 13–24, Sept.–Oct. 1998. (A link to this article is provided on our website, www.mhhe.com/hillier.)

You will see later that the spreadsheet is made easier to interpret by using range names. **A range name** is a descriptive name given to a block of cells that immediately identifies what is there. Thus, the data cells in the Wyndor problem are given the range names UnitProfit (C4:D4), HoursUsedPerBatchProduced (C7:D9), and HoursAvailable (G7:G9). Note that no spaces are allowed in a range name so each new word begins with a capital letter. To enter a range name, first select the range of cells, then click in the name box on the left of the formula bar above the spreadsheet and type a name.

Three questions need to be answered to begin the process of using the spreadsheet to formulate a linear programming model for the problem.

1. What are the *decisions* to be made? For this problem, the necessary decisions are the *production rates* (number of batches produced per week) for the two new products.
2. What are the *constraints* on these decisions? The constraints here are that the number of hours of production time used per week by the two products in the respective plants cannot exceed the number of hours available.
3. What is the overall *measure of performance* for these decisions? Wyndor's overall measure of performance is the *total profit* per week from the two products, so the *objective* is to *maximize* this quantity.

Figure 3.15 shows how these answers can be incorporated into the spreadsheet. Based on the first answer, the *production rates* of the two products are placed in cells C12 and D12 to locate them in the columns for these products just under the data cells. Since we don't know yet what these production rates should be, they are just entered as zeroes at this point. (Actually, any trial solution can be entered, although *negative* production rates should be excluded since they are impossible.) Later, these numbers will be changed while seeking the best mix of production rates. Therefore, these cells containing the decisions to be made are called **changing cells**. To highlight the changing cells, they are shaded and have a border. (In the spreadsheet files contained in OR Courseware, the changing cells

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Profit Per Batch ($000)	3	5			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	0	<=	4
8		Plant 2	0	2	0	<=	12
9		Plant 3	3	2	0	<=	18
10							
11			Doors	Windows			Total Profit ($000)
12		Batches Produced	0	0			0

FIGURE 3.15
The complete spreadsheet for the Wyndor problem with an initial trial solution (both production rates equal to zero) entered into the changing cells (C12 and D12).

appear in bright yellow on a color monitor.) The changing cells are given the range name BatchesProduced (C12:D12).

Using the answer to question 2, the total number of hours of production time used per week by the two products in the respective plants is entered in cells E7, E8, and E9, just to the right of the corresponding data cells. The Excel equations for these three cells are

E7 = C7*C12 + D7*D12

E8 = C8*C12 + D8*D12

E9 = C9*C12 + D9*D12

where each asterisk denotes multiplication. Since each of these cells provides output that depends on the changing cells (C12 and D12), they are called **output cells.**

Notice that each of the equations for the output cells involves the sum of two products. There is a function in Excel called SUMPRODUCT that will sum up the product of each of the individual terms in two different ranges of cells when the two ranges have the same number of rows and the same number of columns. Each product being summed is the product of a term in the first range and the term in the corresponding location in the second range. For example, consider the two ranges, C7:D7 and C12:D12, so that each range has one row and two columns. In this case, SUMPRODUCT (C7:D7, C12:D12) takes each of the individual terms in the range C7:D7, multiplies them by the corresponding term in the range C12:D12, and then sums up these individual products, as shown in the first equation above. Using the range name BatchesProduced (C12:D12), the formula becomes SUMPRODUCT (C7:D7, BatchesProduced). Although optional with such short equations, this function is especially handy as a shortcut for entering longer equations.

Next, ≤ signs are entered in cells F7, F8, and F9 to indicate that each total value to their left cannot be allowed to exceed the corresponding number in column G. The spreadsheet still will allow you to enter trial solutions that violate the ≤ signs. However, these ≤ signs serve as a reminder that such trial solutions need to be rejected if no changes are made in the numbers in column G.

Finally, since the answer to the third question is that the overall measure of performance is the total profit from the two products, this profit (per week) is entered in cell G12. Much like the numbers in column E, it is the sum of products,

G12 = SUMPRODUCT (C4:D4, C12:D12)

Utilizing range names of TotalProfit (G12), ProfitPerBatch (C4:D4), and BatchesProduced (C12:D12), this equation becomes

TotalProfit = SUMPRODUCT (ProfitPerBatch, BatchesProduced)

This is a good example of the benefit of using range names for making the resulting equation easier to interpret. Rather than needing to refer to the spreadsheet to see what is in cells G12, C4:D4, and C12:D12, the range names immediately reveal what the equation is doing.

TotalProfit (G12) is a special kind of output cell. It is the particular cell that is being targeted to be made as large as possible when making decisions regarding production rates. Therefore, TotalProfit (G12) is referred to as the **objective cell**. The objective cell is shaded darker than the changing cells and is further distinguished by having a heavy border. (In the spreadsheet files contained in OR Courseware, this cell appears in orange on a color monitor.)

The bottom of Fig. 3.16 summarizes all the formulas that need to be entered in the Hours Used column and in the Total Profit cell. Also shown is a summary of the range names (in alphabetical order) and the corresponding cell addresses.

This completes the formulation of the spreadsheet model for the Wyndor problem.

With this formulation, it becomes easy to analyze any trial solution for the production rates. Each time production rates are entered in cells C12 and D12, Excel immediately calculates the output cells for hours used and total profit. However, it is not necessary to use trial and error. We shall describe next how Solver can be used to quickly find the optimal solution.

Using Solver to Solve the Model

Excel includes a tool called **Solver** that uses the simplex method to find an optimal solution. ASPE (an Excel add-in available in your OR Courseware) includes a more advanced version of Solver that can also be used to solve this same problem. ASPE's Solver will be described in the next subsection.

■ **FIGURE 3.16**
The spreadsheet model for the Wyndor problem, including the formulas for the objective cell TotalProfit (G12) and the other output cells in column E, where the goal is to maximize the objective cell.

	A	B	C	D	E	F	G
1		**Wyndor Glass Co. Product-Mix Problem**					
2							
3			Doors	Windows			
4		Profit Per Batch ($000)	3	5			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	0	<=	4
8		Plant 2	0	2	0	<=	12
9		Plant 3	3	2	0	<=	18
10							
11			Doors	Windows			Total Profit ($000)
12		Batches Produced	0	0			0

Range Name	Cells
BatchesProduced	C12:D12
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerBatchProduced	C7:D9
ProfitPerBatch	C4:D4
TotalProfit	G12

	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7,BatchesProduced)
8	=SUMPRODUCT(C8:D8,BatchesProduced)
9	=SUMPRODUCT(C9:D9,BatchesProduced)

	G
11	Total Profit
12	=SUMPRODUCT(ProfitPerBatch,BatchesProduced)

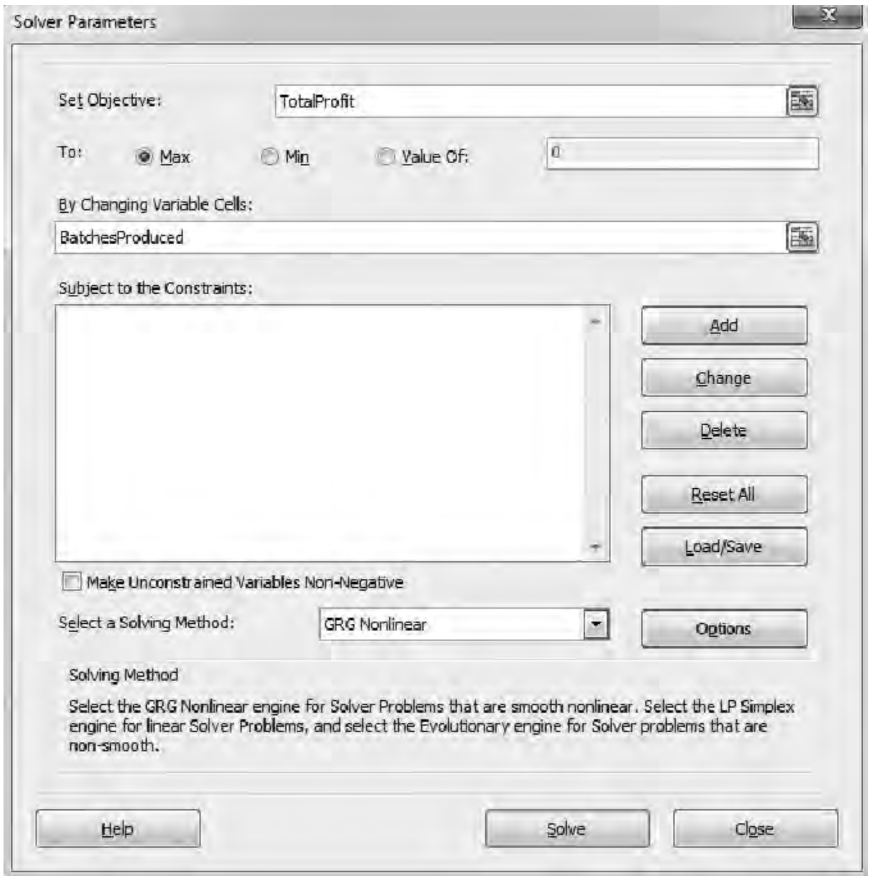
To access the standard Solver for the first time, you need to install it. Click the Office Button, choose Excel Options, then click on Add-Ins on the left side of the window, select Manage Excel Add-Ins at the bottom of the window, and then press the Go button. Make sure Solver is selected in the Add-Ins dialog box, and then it should appear on the Data tab. For Excel 2011 (for the Mac), choose Add-Ins from the Tools menu and make sure that Solver is selected.

To get started, an arbitrary trial solution has been entered in Fig. 3.16 by placing zeroes in the changing cells. Solver will then change these to the optimal values after solving the problem.

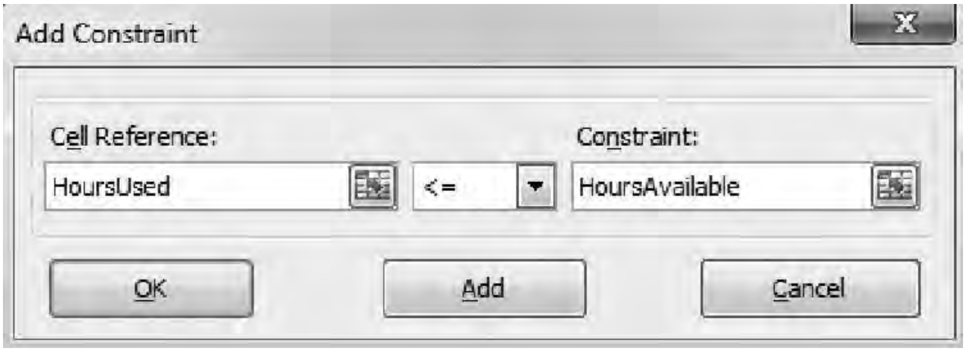
This procedure is started by clicking on the Solver button on the Data tab. The Solver dialog box is shown in Fig. 3.17.

Before Solver can start its work, it needs to know exactly where each component of the model is located on the spreadsheet. The Solver dialog box is used to enter this information. You have the choice of typing the range names, typing in the cell addresses, or clicking on the cells in the spreadsheet.[9] Figure 3.17 shows the result of using the first choice, so TotalProfit (rather than G12) has been entered for the objective cell and BatchesProduced (rather than the range C12:D12) has been entered for the changing cells. Since the goal is to maximize the objective cell, Max also has been selected.

FIGURE 3.17
This Solver dialog box specifies which cells in Fig. 3.16 are the objective cell and the changing cells. It also indicates that the objective cell is to be maximized.



[9]If you select cells by clicking on them, they will first appear in the dialog box with their cell addresses and with dollar signs (e.g., $C$9:$D$9). You can ignore the dollar signs. Solver will eventually replace both the cell addresses and the dollar signs with the corresponding range name (if a range name has been defined for the given cell addresses), but only after either adding a constraint or closing and reopening the Solver dialog box.



■ **FIGURE 3.18**
The Add Constraint dialog box after entering the set of constraints, HoursUsed (E7:E9) ≤ HoursAvailable (G7:G9), which specifies that cells E7, E8, and E9 in Fig. 3.16 are required to be less than or equal to cells G7, G8, and G9, respectively.

Next, the cells containing the functional constraints need to be specified. This is done by clicking on the Add button on the Solver dialog box. This brings up the Add Constraint dialog box shown in Fig. 3.18. The ≤ signs in cells F7, F8, and F9 of Fig. 3.16 are a reminder that the cells in HoursUsed (E7:E9) all need to be less than or equal to the corresponding cells in HoursAvailable (G7:G9). These constraints are specified for Solver by entering HoursUsed (or E7:E9) on the left-hand side of the Add Constraint dialog box and HoursAvailable (or G7:G9) on the right-hand side. For the sign between these two sides, there is a menu to choose between <= (less than or equal), =, or >= (greater than or equal), so <= has been chosen. This choice is needed even though ≤ signs were previously entered in column F of the spreadsheet because Solver only uses the functional constraints that are specified with the Add Constraint dialog box.

If there were more functional constraints to add, you would click on Add to bring up a new Add Constraint dialog box. However, since there are no more in this example, the next step is to click on OK to go back to the Solver dialog box.

Before asking Solver to solve the model, two more steps need to be taken. We need to tell Solver that non-negativity constraints are needed for the changing cells to reject negative production rates. We also need to specify that this is a *linear* programming problem so the simplex method can be used. This is demonstrated in Figure 3.19, where the *Make Unconstrained Variables Non-Negative* option has been checked and the *Solving Method* chosen is *Simplex LP* (rather than *GRG Nonlinear* or *Evolutionary*, which are used for solving nonlinear problems). The Solver dialog box shown in this figure now summarizes the complete model.

Now you are ready to click on Solve in the Solver dialog box, which will start the process of solving the problem in the background. After a fraction of a second (for a small problem), Solver will then indicate the outcome. Typically, it will indicate that it has found an optimal solution, as specified in the Solver Results dialog box shown in Fig. 3.20. If the model has no feasible solutions or no optimal solution, the dialog box will indicate that instead by stating that "Solver could not find a feasible solution" or that "The Objective Cell values do not converge." The dialog box also presents the option of generating various reports. One of these (the Sensitivity Report) will be discussed later in Secs. 4.7 and 7.3.

After solving the model, Solver replaces the original numbers in the changing cells with the optimal numbers, as shown in Fig. 3.21. Thus, the optimal solution is to produce two batches of doors per week and six batches of windows per week, just as was found by the graphical method in Sec. 3.1. The spreadsheet also indicates the corresponding number in the objective cell (a total profit of $36,000 per week), as well as the numbers in the output cells HoursUsed (E7:E9).

FIGURE 3.19
The Solver dialog box after specifying the entire model in terms of the spreadsheet.

FIGURE 3.20
The Solver Results dialog box that indicates that an optimal solution has been found.

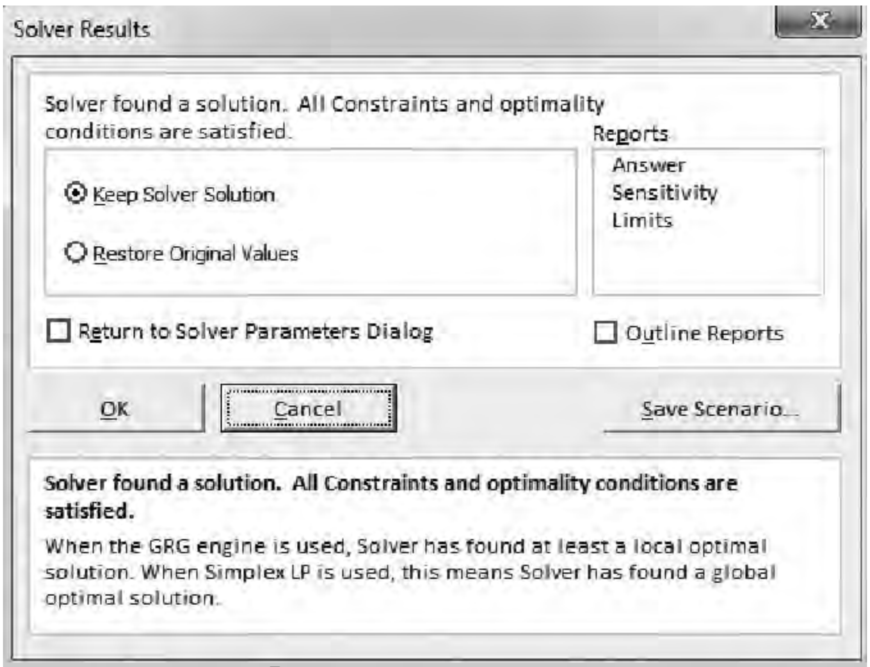


FIGURE 3.21
The spreadsheet obtained after solving the Wyndor problem.

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Profit Per Batch ($000)	3	5			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit ($000)
12		Batches Produced	2	6			36

Solver Parameters
Set Objective Cell: TotalProfit
To: Max
By Changing Variable Cells:
BatchesProduced
Subject to the Constraints:
HoursUsed<= HoursAvailable
Solver Options:
Make Variables Nonnegative
Solving Method: Simplex LP

	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7,BatchesProduced)
8	=SUMPRODUCT(C8:D8,BatchesProduced)
9	=SUMPRODUCT(C9:D9,BatchesProduced)

	G
11	Total Profit
12	=SUMPRODUCT(ProfitPerBatch,BatchesProduced)

Range Name	Cells
BatchesProduced	C12:D12
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerBatchProduced	C7:D9
ProfitPerBatch	C4:D4
TotalProfit	G12

At this point, you might want to check what would happen to the optimal solution if any of the numbers in the data cells were changed to other possible values. This is easy to do because Solver saves all the addresses for the objective cell, changing cells, constraints, and so on when you save the file. All you need to do is make the changes you want in the data cells and then click on Solve in the Solver dialog box again. (Sections 4.7 and 7.3 will focus on this kind of *sensitivity analysis,* including how to use Solver's Sensitivity Report to expedite this type of what-if analysis.)

To assist you with experimenting with these kinds of changes, your OR Courseware includes Excel files for this chapter (as for others) that provide a complete formulation and solution of the examples here (the Wyndor problem and the ones in Sec. 3.4) in a spreadsheet format. We encourage you to "play" with these examples to see what happens with different data, different solutions, and so forth. You might also find these spreadsheets useful as templates for solving homework problems.

In addition, we suggest that you use this chapter's Excel files to take a careful look at the spreadsheet formulations for some of the examples in Sec. 3.4. This will demonstrate how to formulate linear programming models in a spreadsheet that are larger and more complicated than for the Wyndor problem.

You will see other examples of how to formulate and solve various kinds of OR models in a spreadsheet in later chapters. The supplementary chapters on the book's website also include a complete chapter (Chap. 21) that is devoted to the art of modeling in spreadsheets. That chapter describes in detail both the general process and the basic guidelines for building a spreadsheet model. It also presents some techniques for debugging such models.

APPENDIX E

MICROSOFT EXCEL AND SOLVER

In this appendix, we demonstrate the procedure used to solve a linear programming problem with Microsoft Excel and Solver. We assume that the reader is familiar with the standard spreadsheet techniques and formulas.

Implementing Microsoft Excel and Solver to solve a linear programming problem is accomplished in four basic steps:

1. The data for the problem are entered on the spreadsheet.
2. A representation of the mathematical model for the problem is constructed on the spreadsheet, usually below the data section.
3. The representation of the problem is transferred to Solver.
4. Using Solver, the problem is solved.

Note that the problem is defined on the spreadsheet in the first two steps and that Solver is brought into the solution process only in the last two steps. We illustrate these steps in detail with the following example.

Example E.1. Division P is responsible for the manufacture of two components of the parent company's final product. The division manager has available four different processes to produce the two parts. Each process uses varying amounts of labor and two raw materials, with inputs, outputs, and cost of 1 hr operation of each process given in the following table.

		Process 1	*Process 2*	*Process 3*	*Process 4*
Input	*Labor (worker-hrs)*	8	10	6	12
	Material A (lb)	160	100	200	75
	Material B (lb)	30	35	60	80
Output	*Units of Part 1*	35	45	70	0
	Units of Part 2	55	42	0	90
Cost	*($/hr)*	400	575	620	590

Each week the division is responsible for producing at least 1300 units of Part 1 and 2600 units of Part 2. The division manager has at her disposal weekly up to 2.1 tons of Raw Material A, 1 ton of Raw Material B, and 450 hr of labor. The manager

An Introduction to Linear Programming and Game Theory, Third Edition. By P. R. Thie and G. E. Keough.

can also purchase any number of units of Part 2 from an independent supplier at \$18/unit. To determine the minimum cost of the weekly operation, the manager defines variables x_i = number of hours that Process i is used, $i = 1,2,3,4$, and x_5 = number of units of Part 2 purchased from the outside vendor, and formulates the following linear programming problem:

$$
\begin{array}{l}
\text{Minimize } 400x_1 + 575x_2 + 620x_3 + 590x_4 + 18x_5 \\
\text{subject to} \\
\begin{array}{rcrcrcrcrcll}
8x_1 & + & 10x_2 & + & 6x_3 & + & 12x_4 & & & \le & 450 & \textit{Labor (hr)} \\
160x_1 & + & 100x_2 & + & 200x_3 & + & 75x_4 & & & \le & 4200 & \textit{Material A (lb)} \\
30x_1 & + & 35x_2 & + & 60x_3 & + & 80x_4 & & & \le & 2000 & \textit{Material B (lb)} \\
35x_1 & + & 45x_2 & + & 70x_3 & + & 0x_4 & & & \ge & 1300 & \textit{Units of Part 1} \\
55x_1 & + & 42x_2 & + & 0x_3 & + & 90x_4 & + & x_5 & \ge & 2600 & \textit{Units of Part 2}
\end{array} \\
x_1, x_2, x_3, x_4, x_5 \ge 0
\end{array}
\tag{E.1}
$$

Now, with the data and the linear programming problem at hand, we turn to Microsoft Excel. The initial spreadsheet representation for the problem, with steps 1 and 2 already completed, is in Figure E.1. The data are entered in the upper half of the spreadsheet, as the reader can see. The values of all the coefficients and constant terms of (E.1) are contained in the tables, and the rows, columns, and cells are labeled for easy identification.

	A	B	C	D	E	F	G
1	**Division P**						
2				**Process**			
3		**Input**	1	2	3	4	**Limit**
4		Labor (hr)	8	10	6	12	450
5		Material A (lb)	160	100	200	75	4200
6		Material B (lb)	30	35	60	80	2000
7		**Output**					**# Required**
8		# units Part 1	35	45	70	0	1300
9		# units Part 2	55	42	0	90	2600
10		**Cost ($/hr)**	$400	$575	$620	$590	
11		Part 2 vendor cost/unit -->		$18			
12							
13				**Variables**			
14		Process #	1	2	3	4	
15		Hours used					
16		# Units Part 2 purchased -->					
17							
18		**Minimize cost**					
19							
20		**Constraints**	LHS		RHS		
21		Labor	0	≤	450		
22		Material A	0	≤	4200		
23		Material B	0	≤	2000		
24		Part 1	0	≥	1300		
25		Part 2	0	≥	2600		

Figure E.1

	A	B	C	D	E	F	G
1	Division P						
2				Process			
3		Input	1	2	3	4	Limit
4		Labor (hr)	8	10	6	12	450
5		Material A (lb)	160	100	200	75	4200
6		Material B (lb)	30	35	60	80	2000
7		Output					# Required
8		# units Part 1	35	45	70	0	1300
9		# units Part 2	55	42	0	90	2600
10		Cost ($/hr)	400	575	620	590	
11			Part 2 vendor cost/unit -->	18			
12							
13				Variables			
14		Process #	1	2	3	4	
15		Hours used					
16			# Units Part 2 purchased -->				
17							
18		Minimize cost	=SUMPRODUCT(C10:F10,C15:F15)+D11*D16				
19							
20		Constraints	LHS		RHS		
21		Labor	=SUMPRODUCT(C4:F4,C$15:F$15)	≤	=G4		
22		Material A	=SUMPRODUCT(C5:F5,C$15:F$15)	≤	=G5		
23		Material B	=SUMPRODUCT(C6:F6,C$15:F$15)	≤	=G6		
24		Part 1	=SUMPRODUCT(C8:F8,C$15:F$15)	≥	=G8		
25		Part 2	=SUMPRODUCT(C9:F9,C$15:F$15)+D16	≥	=G9		

Figure E.2

The representation of the actual programming problem of (E.1) is contained in the lower half of the spreadsheet. The construction of this representation consists of three parts.

(p1) The designation of the cells to be used as placeholders for the variables (here cells C15:F15 and D16), the objective function (cell C18), the left-hand sides of the constraints (cells C21:C25), and the right-hand sides of the constraints (cells E21:E25).

(p2) The entering of the appropriate formulas in the objective function and constraints cells, usually through the use of Microsoft Excel's Formula Bar. The region of cells containing formulas for this example (columns C through F, rows 18 through 25) are shown in Figure E.2. Microsoft Excel's SUMPRODUCT function (read "dot product of row vectors" if you wish) is especially helpful in expressing the linear forms of mathematical programming problems, and frequently the formulas can be effectively drag-copied.

(p3) The completion of the listing of the constraints, designating for each constraint the relationship between the left-hand and right-hand sides (cells D21:D25).

The last two steps in solving the problem involve Solver. Clicking on Solver in the Tools pull-down menu superimposes the Solver Parameters window (shown in Figure E.3) on the initial spreadsheet. In this window we enter the spreadsheet locations of the components of the problem to be solved. To be designated in the window are the locations of the cells in the spreadsheet containing the following:

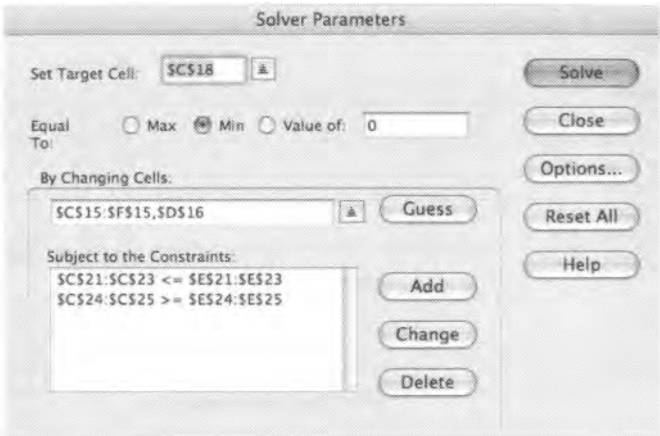


Figure E.3

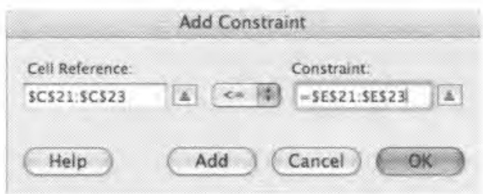


Figure E.4

(s1) The Target Cell, that is, the cell containing the objective function formula (with auxiliary buttons for designating the goal: to maximize or to minimize).

(s2) The Changing Cells, that is, the cells designated for the decision variables.

(s3) The Constraints Cells, both left- and right-hand sides and the type of the constraint. These are added, adjusted, or deleted in the "Subject to the Constraints" area in the lower, left of the Solver Parameters window, utilizing the corresponding pop-up subwindow (the Add Constraint subwindow is shown in Figure E.4). As the reader will see, all the appropriate assignments are in place in the Solver Parameters window of Figure E.3.)

After these steps are completed, a click on the Options button in the Solver Parameters window brings the Solver Options window to the screen, as displayed in Figure E.5. Here, for a linear programming problem we check the "Assume Linear Model" box; and checking the "Assume Non-Negative" box eliminates the need to enter in the constraints set window the nonnegativity restrictions on the variables (if called for in the problem).

That completes the entering of the specifics of the problem into Solver. Clicking the Solve button in the Solver Parameters window will now generate the "Solver Results" window displayed in Figure E.6. Since a solution exists for this problem, the Solver Results window shows the message "Solver found a solution. All constraints and optimality conditions are satisfied." The solution values for the variables, objec-

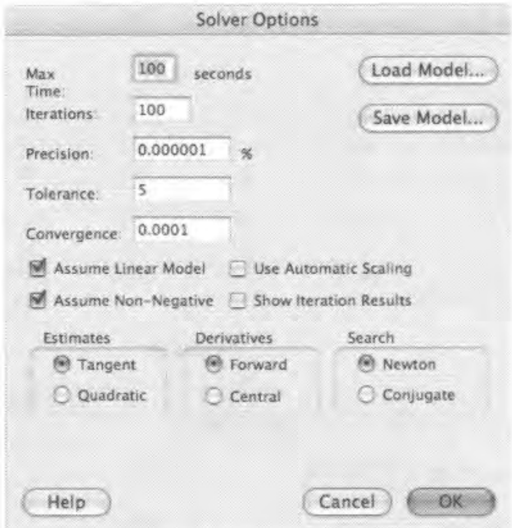


Figure E.5

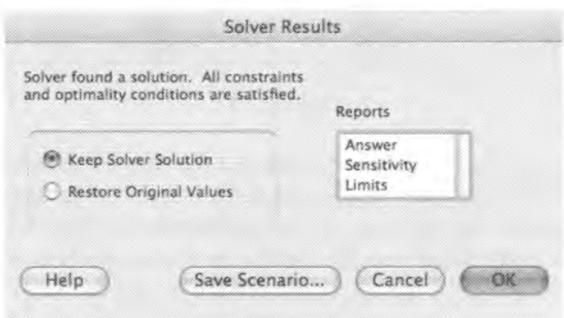


Figure E.6

tive function, and constraints will be displayed on the original spreadsheet, as seen in Figure E.7. The user here also has the option of generating the associated Sensitivity Report by clicking the corresponding word in the Reports window. The nature of this report is discussed at some length in Sections 5.1 and 5.3.

Two other messages can be displayed when the Solver Results window appears, indicating either that the objective function is unbounded ("The Set Cell values do not converge") or that the problem has no feasible solution ("Solver could not find a feasible solution"). *One must carefully read the message in the Solver Results Window before clicking OK to dismiss it*, since each of these outcomes may modify the data on the original spreadsheet; the hurried user might then unwittingly believe that a solution has been found upon returning to the spreadsheet.

We close with some helpful comments on using Solver and Microsoft Excel:

	A	B	C	D	E	F	G
1	**Division P**						
2				**Process**			
3		**Input**	1	2	3	4	**Limit**
4		Labor (hr)	8	10	6	12	450
5		Material A (lb)	160	100	200	75	4200
6		Material B (lb)	30	35	60	80	2000
7		**Output**					**# Required**
8		# units Part 1	35	45	70	0	1300
9		# units Part 2	55	42	0	90	2600
10		**Cost ($/hr)**	$400	$575	$620	$590	
11		Part 2 vendor cost/unit -->		$18			
12							
13				**Variables**			
14		Process #	1	2	3	4	
15		Hours used	4.82338	25.13737	0	12.19363	
16		# Units Part 2 purchased -->		181.51766			
17							
18		***Minimize cost***	$26,845				
19							
20		**Constraints**	LHS		RHS		
21		Labor	436	≤	450		
22		Material A	4200	≤	4200		
23		Material B	2000	≤	2000		
24		Part 1	1300	≥	1300		
25		Part 2	2600	≥	2600		

Figure E.7

1. A factor to be considered when laying out the data tables is that the use of the SUMPRODUCT function requires that the arrays being combined flow in the same direction. For example, in the spreadsheet of Figure E.1, the variable cells and their associated coefficients in the constraints both read horizontally, allowing for the easy use of SUMPRODUCT. On the other hand, you may want to make layout adjustments to facilitate the use of the SUMPRODUCT (see, for example, Figure 8.10 of Section 8.4 on page 335, where the variable cells are placed vertically to accommodate the data table structure).
2. Placing the characters ($\leq$) and ($\geq$) in Column D of the initial spreadsheet to indicate the direction of the inequality in each of the five constraints provides only a (very helpful) visual aid. (*The entry of these characters is system-dependent; you may instead prefer to write simply the two-character sequences* $<=$ *or* $>=$.) Solver makes no use of these entries, however; the appropriate inequality relations must still be entered directly in the Add Constraints window in step (s3) above.
3. The solution to (E.1) on the spreadsheet in Figure E.7 calls for nonintegral values for the variables. If integral values are required, one could, on the spreadsheet, round off the value of each of the variables to the nearest integer and then note the feasibility or nonfeasibility of this set of integers using the spreadsheet's adjusted values for the left-hand sides of the constraints. Here, in fact, the results would show that feasibility is maintained for the first four

3.10 SPREADSHEET SOLUTION OF A LINEAR PROGRAMMING PROBLEM

While the simplex method can be used to solve linear programming problems of any size, if we are restricted to working by hand or with LP Assistant, large problems can easily become unmanageable. However, there are many commercial products that can solve large and realistic problems. In this section we demonstrate via examples the use of one such product, Microsoft Excel's spreadsheet tool Solver. A description of this application and an outline of how to use it are presented in Appendix E. Here we present only the final spreadsheet resolution using Solver for three examples. In subsequent chapters we will discuss Solver's associated sensitivity report.

Example 3.10.1. Using units of the component materials A, B, C, and D, Company Zeta produces Products 1, 2, and 3. The input (units of each component material) and profit per unit produced of the products, and the available supplies for the next month of the component materials, are as follows.

	Product			
Component	1	2	3	*Supply (units)*
A	16	30	28	1550
B	24	40	36	2044
C	30	50	32	2438
D	10	20	15	975
Profit/unit	$78	$136	$104	

To determine the optimal production schedule and profit for the next month, the company analyst defines variables x_1, x_2, x_3 to be the number of units of product i to be produced, $i = 1,2,3$, and formulates the following model:

Maximize profit z (in \$), $z = 78x_1 + 136x_2 + 104x_3$
subject to

$$\begin{aligned}
16x_1 + 30x_2 + 28x_3 &\leq 1550\\
24x_1 + 40x_2 + 36x_3 &\leq 2044\\
30x_1 + 50x_2 + 32x_3 &\leq 2438\\
10x_1 + 20x_2 + 15x_3 &\leq 975\\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

The spreadsheet resolution appears in Figure 3.5. Company Zeta's optimal profit for next month is $6,748, attained by making 10 units of Product 1, 37 units of Product 2, and 9 units of Product 3. The component materials constraints show that with this production schedule, surplus units remain only for material A. However, with this, as with any spreadsheet resolution of a problem, much of the action is behind the scenes. For example, besides what is seen on the spreadsheet, formulas define the values of the objective function cell and the cells for the left-hand and right-hand sides of the constraints. Furthermore, beyond the spreadsheet, the actual mathematical problem

	A	B	C	D	E	F
1	**Company Zeta**					
2				**Product**		
3		**Component**	1	2	3	**Supply**
4		A	16	30	28	1550
5		B	24	40	36	2044
6		C	30	50	32	2438
7		D	10	20	15	975
8		**Profit/unit**	$78	$136	$104	
9						
10				**Variables**		
11		Product #	1	2	3	
12		Units made	10	37	9	
13						
14		**Maximize Profit**	$6,748			
15						
16		Comp. Materials	LHS		RHS	
17		A	1522	≤	1550	
18		B	2044	≤	2044	
19		C	2438	≤	2438	
20		D	975	≤	975	

Figure 3.5

is established on the tool Solver, and then Solver is invoked to resolve the problem. This is all explained in Appendix E.

Example 3.10.2 (Similar to Example 2.2.3). A landscaper has two fields to maintain, Field X and Field Y, with each field requiring grass seed mixtures of specified percentages of bluegrass and fescue. To meet these needs, the landscaper has three grass seed blends with which to work. The relevant data are summarized in the following table.

		Bluegrass	*Fescue*	*Cost (cents/lb)*
Composition	*Blend I*	60%	10%	80
	Blend II	20%	50%	95
	Blend III	25%	15%	35
Requirements	*Field X*	$\geq 30\%$	$\geq 10\%$	
	Field Y	$\geq 25\%$	$\geq 45\%$	

The landscaper has an order for 200 lb of seed for Field X and 180 lb of seed for Field Y; and on hand to fill the order there are unlimited amounts of Blends I and II but only 125 lb of Blend III.

To determine the minimum cost to meet these demands, the following model is formulated. Let x_1, x_2, x_3 be the number of pounds of Blends I, II, and III, respectively, used for Field X, and let y_1, y_2, y_3 be the number of pounds of each used for Field Y. The problem:

	A	B	C	D	E	F
1	**Landscaper**					
2				**Composition Data for Blends**		
3			Bluegrass	Fescue	Cost (cents/lb)	
4		Blend I	60%	10%	$0.80	
5		Blend II	20%	50%	$0.95	
6		Blend III	25%	15%	$0.35	
7						
8				**Min Requirements Data for Fields**		
9			Bluegrass	Fescue	Pounds	
10		Field X	30%	10%	200	
11		Field Y	25%	45%	180	
12						
13			**Variables (lb by Blend and Field)**			
14			Field X	Field Y		
15		Blend I	75	22.5		
16		Blend II	0	157.5		
17		Blend III	125	0.00		
18						
19		**Minimize Cost**	$271.38			
20						
21		**Constraints**	LHS		RHS	
22		Field X Bluegrass	76.25	≥	60	
23		Field X Fescue	26.25	≥	20	
24		Field X Total	200	=	200	
25		Field Y Bluegrass	45	≥	45	
26		Field Y Fescue	81	≥	81	
27		Field Y Total	180	=	180	
28		Blend III Maximum	125	≤	125	

Figure 3.6

To minimize the function $(80x_1 + 95x_2 + 35x_3) + (80y_1 + 95y_2 + 35y_3)$
subject to

$$
\begin{array}{ll}
x_1 + x_2 + x_3 = 200 & y_1 + y_2 + y_3 = 180 \\
.6x_1 + .2x_2 + .25x_3 \geq 0.3(200) = 60 & .6y_1 + .2y_2 + .25y_3 \geq .25(180) = 45 \\
.1x_1 + .5x_2 + .15x_3 \geq 0.1(200) = 20 & .1y_1 + .5y_2 + .15y_3 \geq .45(180) = 81 \\
\multicolumn{2}{c}{x_3 + y_3 \leq 125} \\
x_1, x_2, x_3 \geq 0 & y_1, y_2, y_3 \geq 0
\end{array}
$$

The spreadsheet resolution is shown in Figure 3.6. The minimum cost for the landscaper is $271.38, attained by using 75 lb of Blend I and 125 lb of Blend III in preparing the 200-lb mix for Field X and using 22.5 lb of Blend I and 157.5 lb of Blend II in preparing the 180-lb mix for Field Y. All of the available 125 pounds of Blend III are utilized.

This suggests an obvious question. How much money might be saved if more of Blend III were available? The answer to this question, and similar ones, is available from the final tableau of the simplex algorithm resolution of the problem and on

4.3 Examples and Interpretations

In Section 4.1, the dual to a production problem involving profits to be maximized was developed. In this section the dual problems to other specific linear programming examples will be defined and discussed. The examples, using the categories of Chapter 2, are from the classes of blending problems, production problems (minimizing costs while meeting given demands), and transportation problems. Additional examples are contained in the problems at the end of this section.

Example 4.3.1 (A Blending Problem). The diet problems that we have already seen lead to dual problems that have a standard but still extremely interesting interpretation. Consider, for example, the situation described in Example 2.2.1 on page 10 of the farmer wishing to feed her stock. The farmer's problem was to determine a diet using two feeds that minimized cost and satisfied three nutritional requirements. Here, letting x_1 and x_2 denote the amounts in pounds of Feeds 1 and 2 to use, respectively, the mathematical problem was to

$$\begin{aligned}&\text{Minimize } 10x_1 + 4x_2 \\ &\text{subject to} \\ &3x_1 + 2x_2 \geq 60 \\ &7x_1 + 2x_2 \geq 84 \\ &3x_1 + 6x_2 \geq 72 \\ &x_1, x_2 \geq 0\end{aligned} \tag{4.3.1}$$

The three inequalities in the system of constraints result from the requirement that the diet provide specified amounts of the nutritional elements A, B, and C. The dual to this problem is the problem of

$$\begin{aligned}&\text{Maximizing } 60y_1 + 84y_2 + 72y_3 \\ &\text{subject to} \\ &3y_1 + 7y_2 + 3y_3 \leq 10 \\ &2y_1 + 2y_2 + 6y_3 \leq 4 \\ &y_1, y_2, y_3 \geq 0\end{aligned} \tag{4.3.2}$$

To provide an interpretation of the dual, consider the problem of a traveling salesman dealing in nutrition tablets for cattle. Suppose the salesman has to offer the farmer three types of pure tablet: one type containing 1 unit of nutritional element A and nothing else, one containing 1 unit of B and nothing else, and the last containing 1 unit of C and nothing else. Now the salesman hopes to convince the farmer that it is to her advantage to nourish her cattle by using these tablets instead of any combination of Feeds 1 and 2. Although the farmer is probably somewhat set in her ways, the salesman believes that due to the problems of maintaining a small farm today, he can still appeal to her frugality. Thus the salesman attempts to set the prices for the three types of tablets in such a way that the tablets can compete favorably with the two feeds and he can realize the greatest income. To do this, he lets y_1, y_2, and y_3

Table 4.2

	y_1	y_2	y_3	y_4	y_5	
y_4	3	7	3	1	0	10
y_5	2	2	6	0	1	4
	-60	-84	-72	0	0	0
y_4	0	4	-6	1	$-\frac{3}{2}$	4
y_1	1	1	3	0	$\frac{1}{2}$	2
	0	-24	108	0	30	120
y_2	0	1	$-\frac{3}{2}$	$\frac{1}{4}$	$-\frac{3}{8}$	1
y_1	1	0	$\frac{9}{2}$	$-\frac{1}{4}$	$\frac{7}{8}$	1
	0	0	72	6	21	144

denote the cost in cents to the farmer of one tablet of nutritional elements A, B, and C, respectively.

Now 1 lb of Feed 1 provides 3, 7, and 3 units of A, B, and C, respectively, and costs 10 cents. To replace 1 lb of this feed with tablets, the farmer would need three tablets each of the first and third types and seven of the second type. This would cost $3y_1 + 7y_2 + 3y_3$ cents and so, to be competitive, the salesman must have

$$3y_1 + 7y_2 + 3y_3 \leq 10$$

Similarly, 1 lb of Feed 2 provides 2, 2, and 6 units of A, B, and C, respectively, and cost 4 cents. Thus we have the inequality

$$2y_1 + 2y_2 + 6y_3 \leq 4$$

Since the farmer has determined that the daily requirements of elements A, B, and C are 60, 84, and 72 units, respectively, the cost of meeting these requirements by using the tablets would be $60y_1 + 84y_2 + 72y_3$. Thus the salesman wishes to maximize this function subject to the above two inequalities. This problem is precisely the dual of the original problem.

Being a former mathematician, the salesman does not stop here but sets out to solve the linear programming problem (4.3.2). Adding two slack variables and using the simplex method, he generates the tableaux of Table 4.2. From the final tableau, in which we see that $y_1 = y_2 = 1$ and $y_3 = 0$, the salesman notes that he should charge the farmer 1 cent for each of the tablets of A and B and nothing for the tablets of C ("Place your order today and receive the C tablets at no extra charge"), and in doing this, he will realize his maximum income of \$1.44. Observe that this maximum income of \$1.44 equals the minimum cost to the farmer of an adequate diet using Feeds 1 and 2, as determined in Section 2.2.

In the next section, we will show that the above result is not just coincidental. The Duality Theorem, as we will see, states that the min problem of (4.3.1) and

Example 4.4.2. Consider the linear programming problem of

$$\begin{array}{l} \text{Minimizing } 20x_1 + 15x_2 + 54x_3 \\ \text{subject to} \\ \begin{array}{rcrcrcr} x_1 & - & 2x_2 & + & 6x_3 & \geq & 30 \\ & & x_2 & + & 2x_3 & \geq & 6 \\ 2x_1 & & & - & 3x_3 & \geq & -5 \\ x_1 & - & x_2 & & & \geq & 18 \end{array} \\ x_1, x_2, x_3 \geq 0 \end{array}$$

To solve this problem using the simplex method, we would first add 4 slack variables, then 3 artificial variables (the slack variable in the third constraint could serve as a basic variable), and use the full two stages of the algorithm on the resulting problem of 4 constraints and 10 variables. However, the dual to this problem is to

$$\begin{array}{l} \text{Maximize } 30y_1 + 6y_2 - 5y_3 + 18y_4 \\ \text{subject to} \\ \begin{array}{rcrcrcrcr} y_1 & & & + & 2y_3 & + & y_4 & \leq & 20 \\ -2y_1 & + & y_2 & & & - & y_4 & \leq & 15 \\ 6y_1 & + & 2y_2 & - & 3y_3 & & & \leq & 54 \end{array} \\ y_1, y_2, y_3, y_4 \geq 0 \end{array}$$

Applying the simplex algorithm to this dual problem is somewhat easier. Adding three slack variables and solving, we have the tableaux of Table 4.3. The maximum value of the objective function $30y_1 + 6y_2 - 5y_3 + 18y_4$ is 522, and therefore the minimum value of the objective function of the original problem also is 522. Moreover, from the bottom row of the final tableau, we see that the point $(18, 0, 3)$ is an optimal solution point to the original problem. (Of course, the application of the simplex algorithm to the dual of the minimization problem is facilitated here by the fact that the coefficients in the original objective function, 20, 15, and 54, are all nonnegative. If this had not been the case, computing the solution to the dual with the simplex algorithm would also have required the use of artificial variables.)

These observations suggest a general question. If we solve any linear programming problem with a finite optimal solution using the simplex algorithm, can we always find in the final tableau an optimal solution point to the dual? We address this issue in the following examples, considering first the resolution of a minimization problem.

Example 4.4.3. Consider the problem of Example 4.3.1 of

$$\begin{array}{l} \text{Minimizing } 10x_1 + 4x_2 \\ \text{subject to} \\ 3x_1 + 2x_2 \geq 60 \\ 7x_1 + 2x_2 \geq 84 \\ 3x_1 + 6x_2 \geq 72 \\ x_1, x_2 \geq 0 \end{array}$$

3.5-2.* You are given the following data for a linear programming problem where the objective is to maximize the profit from allocating three resources to two nonnegative activities.

Resource	Resource Usage per Unit of Each Activity: Activity 1	Activity 2	Amount of Resource Available
1	2	1	10
2	3	3	20
3	2	4	20
Contribution per unit	\$20	\$30	

Contribution per unit = profit per unit of the activity.

(a) Formulate a linear programming model for this problem.
D,I **(b)** Use the graphical method to solve this model.
(c) Display the model on an Excel spreadsheet.
(d) Use the spreadsheet to check the following solutions: $(x_1, x_2) = (2, 2), (3, 3), (2, 4), (4, 2), (3, 4), (4, 3)$. Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?
C **(e)** Use the Excel Solver to solve the model by the simplex method.

3.5-3. Ed Butler is the production manager for the Bilco Corporation, which produces three types of spare parts for automobiles. The manufacture of each part requires processing on each of two machines, with the following processing times (in hours):

Machine	Part: A	B	C
1	0.02	0.03	0.05
2	0.05	0.02	0.04

Each machine is available 40 hours per month. Each part manufactured will yield a unit profit as follows:

	Part: A	B	C
Profit	\$300	\$250	\$200

Ed wants to determine the mix of spare parts to produce in order to maximize total profit.
(a) Formulate a linear programming model for this problem.
(b) Display the model on an Excel spreadsheet.
(c) Make three guesses of your own choosing for the optimal solution. Use the spreadsheet to check each one for feasibility and, if feasible, to find the value of the objective function. Which feasible guess has the best objective function value?
(d) Use the Excel Solver to solve the model by the simplex method.

3.5-4. You are given the following data for a linear programming problem where the objective is to minimize the cost of conducting two nonnegative activities so as to achieve three benefits that do not fall below their minimum levels.

Benefit	Benefit Contribution per Unit of Each Activity: Activity 1	Activity 2	Minimum Acceptable Level
1	5	3	60
2	2	2	30
3	7	9	126
Unit cost	\$60	\$50	

(a) Formulate a linear programming model for this problem.
D,J **(b)** Use the graphical method to solve this model.
(c) Display the model on an Excel spreadsheet.
(d) Use the spreadsheet to check the following solutions: $(x_1, x_2) = (7, 7), (7, 8), (8, 7), (8, 8), (8, 9), (9, 8)$. Which of these solutions are feasible? Which of these feasible solutions has the best value of the objective function?
C **(e)** Use the Excel Solver to solve this model by the simplex method.

3.5-5.* Fred Jonasson manages a family-owned farm. To supplement several food products grown on the farm, Fred also raises pigs for market. He now wishes to determine the quantities of the available types of feed (corn, tankage, and alfalfa) that should be given to each pig. Since pigs will eat any mix of these feed types, the objective is to determine which mix will meet certain nutritional requirements at a *minimum cost.* The number of units of each type of basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirements and feed costs:

Nutritional Ingredient	Kilogram of Corn	Kilogram of Tankage	Kilogram of Alfalfa	Minimum Daily Requirement
Carbohydrates	90	20	40	200
Protein	30	80	60	180
Vitamins	10	20	60	150
Cost (¢)	84	72	60	

(a) Formulate a linear programming model for this problem.
(b) Display the model on an Excel spreadsheet.
(c) Use the spreadsheet to check if $(x_1, x_2, x_3) = (1, 2, 2)$ is a feasible solution and, if so, what the daily cost would be for this

diet. How many units of each nutritional ingredient would this diet provide daily?

(d) Take a few minutes to use a trial-and-error approach with the spreadsheet to develop your best guess for the optimal solution. What is the daily cost for your solution?

C **(e)** Use the Excel Solver to solve the model by the simplex method.

3.5-2. (c, e)

Resource	Resource Usage per Unit of Each Activity: Activity 1	Activity 2	Totals		Resource Available
1	2	1	10	≤	10
2	3	3	20	≤	20
3	2	4	20	≤	20
Unit Profit	20	30	\$166.67		
Solution	3.333	3.333			

3.5-5. (a) Minimize $Z = 84C + 72T + 60A$,

subject to

$$90C + 20T + 40A \geq 200$$
$$30C + 80T + 60A \geq 180$$
$$10C + 20T + 60A \geq 150$$

and

$$C \geq 0, \quad T \geq 0, \quad A \geq 0.$$

6.8 PERFORMING SENSITIVITY ANALYSIS ON A SPREADSHEET[7]

With the help of the Excel Solver, spreadsheets provide an alternative, relatively straightforward way of performing much of the sensitivity analysis described in Secs. 6.5–6.7. The spreadsheet approach is basically the same for each of the cases considered in Sec. 6.7 for the types of changes made in the original model. Therefore, we will focus on only the effect of changes in the coefficients of the variables in the objective function (Cases 2*a* and 3 in Sec. 6.7). We will illustrate this effect by making changes in the *original* Wyndor model formulated in Sec. 3.1, where the coefficients of x_1 (number of batches of the new door produced per week) and x_2 (number of batches of the new window produced per week) in the objective function are

$c_1 = 3$ = profit (in thousands of dollars) per batch of the new type of door,
$c_2 = 5$ = profit (in thousands of dollars) per batch of the new type of window.

For your convenience, the spreadsheet formulation of this model (Fig. 3.22) is repeated here as Fig. 6.8. Note that the cells containing the quantities to be changed are ProfitPerBatch (C4:D4). Since the profits in these cells are expressed in dollars, whereas c_1 and c_2 are in units of thousands of dollars, we hereafter will discuss the sensitivity analysis in terms of the changes in the profits shown in these cells instead of changes in c_1 and c_2. To this end, we will denote these profits by

P_D = profit per batch of doors currently entered in cell C4,
P_W = profit per batch of windows currently entered in cell D4.

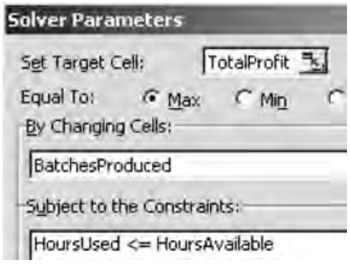
Spreadsheets actually provide three methods of performing sensitivity analysis. One is to check the effect of an individual change in the model by simply making the change on the spreadsheet and re-solving. A second is to systematically generate a table on a single spreadsheet that shows the effect of a series of changes in one or two parameters of the model. A third is to obtain and apply Excel's sensitivity report. We describe each of these methods in turn below.

Checking Individual Changes in the Model

One of the great strengths of a spreadsheet is the ease with which it can be used interactively to perform various kinds of sensitivity analysis. Once the Solver has been set up to obtain an optimal solution, you can immediately find out what would happen if one of the parameters of the model were changed to some other value. All you have to do is make this change on the spreadsheet and then click on the Solve button again.

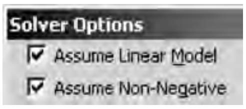
[7] We have written this section in a way that can be understood without first reading any of the preceding sections in this chapter. However, Sec. 4.7 is important background for the latter part of this section.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	$3,000	$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			$36,000



	E
5	Hours
6	Used
7	=SUMPRODUCT(C7:D7,BatchesProduced)
8	=SUMPRODUCT(C8:D8,BatchesProduced)
9	=SUMPRODUCT(C9:D9,BatchesProduced)

	G
11	Total Profit
12	=SUMPRODUCT(ProfitPerBatch,BatchesProduced)



Range Name	Cells
BatchesProduced	C12:D12
HoursAvailable	G7:G9
HoursUsed	E7:E9
HoursUsedPerBatchProduced	C7:D9
ProfitPerBatch	C4:D4
TotalProfit	G12

■ **FIGURE 6.8**
The spreadsheet model and the optimal solution obtained for the original Wyndor problem before performing sensitivity analysis.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	$2,000	$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			$34,000

■ **FIGURE 6.9**
The revised Wyndor problem where the estimate of the profit per batch of doors has been decreased from P_D = \$3,000 to P_D = \$2,000, but no change occurs in the optimal solution for the product mix.

To illustrate, suppose that Wyndor management is quite uncertain about what the profit per batch of doors (P_D) will turn out to be. Although the figure of \$3,000 given in Fig. 6.8 is considered to be a reasonable initial estimate, management feels that the true profit could end up deviating substantially from this figure in either direction. However, the range between P_D = \$2,000 and P_D = \$5,000 is considered fairly likely.

Figure 6.9 shows what would happen if the profit per batch of doors were to drop from P_D = \$3,000 to P_D = \$2,000. Comparing with Fig. 6.8, there is no change at all in

■ **FIGURE 6.10**
The revised Wyndor problem where the estimate of the profit per batch of doors has been increased from P_D = \$3,000 to P_D = \$5,000, but no change occurs in the optimal solution for the product mix.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	\$5,000	\$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			\$40,000

the optimal solution for the product mix. In fact, the *only* changes in the new spreadsheet are the new value of P_D in cell C4 and a decrease of \$2,000 in the total profit shown in cell G12 (because each of the two batches of doors produced per week provides \$1,000 less profit). Because the optimal solution does not change, we now know that the original estimate of P_D = \$3,000 can be considerably *too high* without invalidating the model's optimal solution.

But what happens if this estimate is *too low* instead? Figure 6.10 shows what would happen if P_D were increased to P_D = \$5,000. Again, there is no change in the optimal solution. Therefore, we now know that the range of values of P_D over which the current optimal solution remains optimal (i.e., the *allowable range* discussed in Sec. 6.7) includes the range from \$2,000 to \$5,000 and may extend further.

Because the original value of P_D = \$3,000 can be changed considerably in either direction without changing the optimal solution, P_D is a relatively insensitive parameter. It is not necessary to pin down this estimate with great accuracy in order to have confidence that the model is providing the correct optimal solution.

This may be all the information that is needed about P_D. However, if there is a good possibility that the true value of P_D will turn out to be even outside this broad range from \$2,000 to \$5,000, further investigation would be desirable. How much higher or lower can P_D be before the optimal solution would change?

Figure 6.11 demonstrates that the optimal solution would indeed change if P_D is increased all the way up to P_D = \$10,000. Thus, we now know that this change occurs somewhere between \$5,000 and \$10,000 during the process of increasing P_D.

■ **FIGURE 6.11**
The revised Wyndor problem where the estimate of the profit per batch of doors has been increased from P_D = \$3,000 to P_D = \$10,000, which results in a change in the optimal solution for the product mix.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	\$10,000	\$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	4	<=	4
8		Plant 2	0	2	6	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	4	3			\$55,000

Using the Solver Table to Do Sensitivity Analysis Systematically

To pin down just when the optimal solution will change, we could continue selecting new values of P_D at random. However, a better approach is to systematically consider a range of values of P_D. An Excel add-in developed by Professor Mark Hillier, called the *Solver Table,* is designed to perform just this sort of analysis. It is available to you in your OR Courseware on the book's website. To install it, you need simply to open the Solver Table file in OR Courseware.

The Solver Table is used to show the results in the changing cells and/or certain output cells for various trial values in a data cell. For each trial value in the data cell, Solver is called on to re-solve the problem. Therefore, the Solver Table (or any comparable Excel add-in) provides a systematic way of performing sensitivity analysis and then displaying the results to managers and others who are not familiar with the more technical aspects of sensitivity analysis.

To use the Solver Table, first expand the original spreadsheet (Fig. 6.8) to make a table with headings as shown in Fig. 6.12. In the first column of the table (cells B19:B28), list the trial values for the data cell (the profit per batch of doors), except leave the first row (cell B18) blank. The headings of the next columns specify which output will be evaluated. For each of these columns, use the first row of the table (cells C18:E18) to write an equation that sets the value in each of these cells equal to the relevant changing cell or output cell. In this case, the cells of interest are DoorBatchesProduced (C12), WindowBatchesProduced (D12), and TotalProfit (G12), so the equations for C18:E18 are those shown just below the spreadsheet in Fig. 6.12.

Next, select the entire table by clicking and dragging from cells B18 through E28, and then choose Solver Table from the Add-Ins tab (for Excel 2007) or Tools menu (for earlier versions of Excel), after having installed this Excel add-in provided in your OR Courseware. In the Solver Table dialogue box (as shown at the bottom of Fig. 6.12), indicate the column input cell (C4), which refers to the data cell that is being changed in the first column of the table. Nothing is entered for the row input cell because no row is being used to list the trial values of a data cell in this case.

The Solver Table shown in Fig. 6.13 is then generated automatically by clicking on the OK button. For each trial value listed in the first column of the table for the data cell of interest, Excel re-solves the problem using Solver and then fills in the corresponding values in the other columns of the tables. (The numbers in the first row of the table come from the original solution in the spreadsheet before the original value in the data cell was changed.)

The table reveals that the optimal solution remains the same all the way from $P_D = \$1{,}000$ (and perhaps lower) to $P_D = \$7{,}000$, but that a change occurs somewhere between \$7,000 and \$8,000. We next could systematically consider values of P_D between \$7,000 and \$8,000 to determine more closely where the optimal solution changes. However, this is not necessary since, as discussed a little later, a shortcut is to use the Excel sensitivity report to determine exactly where the optimal solution changes.

Thus far, we have illustrated how to systematically investigate the effect of changing only P_D (cell C4 in Fig. 6.8). The approach is the same for P_W (cell D4). In fact, the Solver Table can be used in this way to investigate the effect of changing *any* single data cell in the model, including any cell in HoursAvailable (G7:G9) or HoursUsedPerBatchProduced (C7:D9).

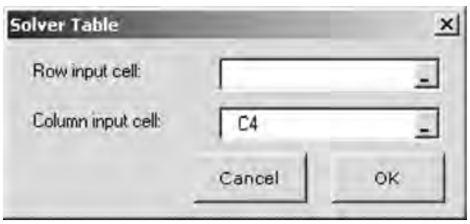
We next will illustrate how to investigate simultaneous changes in two data cells with a spreadsheet, first by itself and then with the help of the Solver Table.

Checking Two-Way Changes in the Model

When using the original estimates for P_D (\$3,000) and P_W (\$5,000), the optimal solution indicated by the model (Fig. 6.8) is heavily weighted toward producing the windows (6 batches per week) rather than the doors (only 2 batches per week). Suppose that

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	$3,000	$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			$36,000
13							
14							
15							
16		Profit Per Batch	Optimal Batches Produced		Total		
17		for Doors	Doors	Windows	Profit		Select these cells (B18:E28), before choosing the Solver Table.
18			2	6	$36,000		
19		$1,000					
20		$2,000					
21		$3,000					
22		$4,000					
23		$5,000					
24		$6,000					
25		$7,000					
26		$8,000					
27		$9,000					
28		$10,000					

	C	D	E
16	Optimal Batches Produced		Total
17	Doors	Windows	Profit
18	=DoorBatchesProduced	=WindowBatchesProduced	=TotalProfit



Range Name	Cells
DoorBatchesProduced	C12
TotalProfit	G12
WIndowBatchesProduced	D12

■ **FIGURE 6.12**
Expansion of the spreadsheet in Fig. 6.8 to prepare for using the Solver Table to show the effect of systematically varying the estimate of the profit per batch of doors in the Wyndor problem.

Wyndor management is concerned about this imbalance and feels that the problem may be that the estimate for P_D is too low and the estimate for P_W is too high. This raises the question: If the estimates are indeed off in these directions, would this lead to a more balanced product mix being the most profitable one? (Keep in mind that it is the *ratio* of P_D to P_W that is relevant for determining the optimal product mix, so having their estimates be off in the *same* direction with little change in this ratio is unlikely to change the optimal product mix).

This question can be answered in a matter of seconds simply by substituting new estimates of the profits per batch in the original spreadsheet in Fig. 6.8 and clicking on the Solve button. Figure 6.14 shows that new estimates of $4,500 for doors and $4,000 for windows causes no change at all in the solution for the optimal product mix. (The total profit does change, but this occurs only because of the changes in the profits per batch.) Would even larger changes in the estimates of profits per batch finally lead to a change

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Profit Per Batch	$3,000	$5,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			$36,000
13							
14							
15							
16		Profit Per Batch	Optimal Batches Produced		Total		
17		for Doors	Doors	Windows	Profit		
18			2	6	$36,000		
19		$1,000	2	6	$32,000		
20		$2,000	2	6	$34,000		
21		$3,000	2	6	$36,000		
22		$4,000	2	6	$38,000		
23		$5,000	2	6	$40,000		
24		$6,000	2	6	$42,000		
25		$7,000	2	6	$44,000		
26		$8,000	4	3	$47,000		
27		$9,000	4	3	$51,000		
28		$10,000	4	3	$55,000		

■ **FIGURE 6.13**
An application of the Solver Table that shows the effect of systematically varying the estimate of the profit per batch for doors in the Wyndor problem.

in the optimal product mix? Figure 6.15 shows that this does happen, yielding a relatively balanced product mix of $(x_1, x_2) = (4, 3)$, when estimates of \$6,000 for doors and \$3,000 for windows are used.

Figures 6.14 and 6.15 don't reveal where the optimal product mix changes as the profit estimates increase from \$4,500 to \$6,000 for doors and decrease from \$4,000 to \$3,000 for windows. We next describe how the Solver Table can systematically help to pin this down better.

■ **FIGURE 6.14**
The revised Wyndor problem where the estimates of the profits per batch of doors and windows have been changed to $P_D =$ \$4,500 and $P_W =$ \$4,000, respectively, but no change occurs in the optimal product mix.

	A	B	C	D	E	F	G
1	Wyndor Glass Co. Product-Mix Problem						
2							
3			Doors	Windows			
4		Profit Per Batch	$4,500	$4,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	2	<=	4
8		Plant 2	0	2	12	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	2	6			$33,000

■ **FIGURE 6.15**
The revised Wyndor problem where the estimates of the profits per batch of doors and windows have been changed to $6,000 and $3,000, respectively, which results in a change in the optimal product mix.

	A	B	C	D	E	F	G
1	**Wyndor Glass Co. Product-Mix Problem**						
2							
3			Doors	Windows			
4		Profit Per Batch	$6,000	$3,000			
5					Hours		Hours
6			Hours Used Per Batch Produced		Used		Available
7		Plant 1	1	0	4	<=	4
8		Plant 2	0	2	6	<=	12
9		Plant 3	3	2	18	<=	18
10							
11			Doors	Windows			Total Profit
12		Batches Produced	4	3			$33,000

Using the Solver Table for Two-Way Sensitivity Analysis

A two-way version of the Solver Table provides a way of systematically investigating the effect if the estimates entered into *two* data cells are inaccurate simultaneously. (However, two is the maximum number of data cells that can be considered simultaneously by the Solver Table.) In this case, the Solver Table shows the results in a single output cell for various trial values in the two data cells.

To illustrate this approach, we again will investigate the effect of increasing P_D and decreasing P_W simultaneously. Before considering the effect on the optimal product mix, we will look at the effect on the total profit. To do this, the Solver Table will be used to show how TotalProfit (G12) in Fig. 6.8 varies over a range of trial values in the two data cells, ProfitPerBatch (C4:D4). For each pair of trial values in these data cells, Solver will be called on to re-solve the problem.

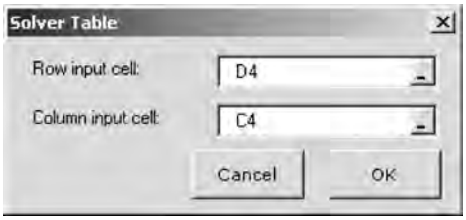
To create a two-way Solver Table for the Wyndor problem, expand the original spreadsheet (Fig. 6.8) to make a table with column and row headings as shown in rows 16–21 of the spreadsheet in Fig. 6.16. In the upper left-hand corner of the table (C17), write an equation (=TotalProfit) that refers to the target cell. In the first column of the table (column C, below the equation in cell C17), insert various trial values for the first data cell of interest (the profit per batch of the doors). In the first row of the table (row 17, to the right of the equation in cell C17), insert various trial values for the second data cell of interest (the profit per batch of the windows).

Next, select the entire table (C17:H21) and choose Solver Table from the Add-Ins tab (for Excel 2007) or Tools menu (for earlier versions of Excel), after having installed this Excel add-in provided in your OR Courseware. In the Solver Table dialogue box (shown at the bottom of Fig. 6.16), indicate which data cells are being changed simultaneously. The column input cell C4 refers to the data cell whose various trial values are listed in the first column of the table (C18:C21), while the row input cell D4 refers to the data cell whose various trial values are listed in the first row of the table (D17:H17).

The Solver Table shown in Fig. 6.17 is then generated automatically by clicking on the OK button. For each pair of trial values for the two data cells, Excel re-solves the problem using Solver and then fills in the total profit in the corresponding spot in the table. (The number in C17 comes from the target cell in the original spreadsheet before the original values in the two data cells are changed.)

Unlike a one-way Solver Table that can show the results of *multiple* changing cells and/or output cells for various trial values of a single data cell, a two-way Solver Table is limited to showing the results in a *single* cell for each pair of trial values in the two data cells of interest.

	A	B	C	D	E	F	G	H	I
1	**Wyndor Glass Co. Product-Mix Problem**								
2									
3			Doors	Windows					
4		Profit Per Batch	$3,000	$5,000					
5					Hours		Hours		
6			Hours Used Per Batch Produced		Used		Available		
7		Plant 1	1	0	2	<=	4		
8		Plant 2	0	2	12	<=	12		
9		Plant 3	3	2	18	<=	18		Select these cells (C17:H21), before choosing the Solver Table.
10									
11			Doors	Windows			Total Profit		
12		Batches Produced	2	6			$36,000		
13									
14									
15									
16		**Total Profit**			Profit Per Batch for Windows				
17			$36,000	$1,000	$2,000	$3,000	$4,000	$5,000	
18			$3,000						
19		Profit Per Batch	$4,000						
20		for Doors	$5,000						
21			$6,000						



	C
17	=TotalProfit

Range Name	Cell
TotalProfit	G12

■ **FIGURE 6.16**
Expansion of the spreadsheet in Fig. 6.8 to prepare for using a two-dimensional Solver Table to show the effect on total profits of systematically varying the estimates of the profits per batch of doors and windows for the Wyndor problem.

■ **FIGURE 6.17**
A two-dimensional application of the Solver Table that shows the effect on the optimal total profit of systematically varying the estimates of the profits per batch of doors and windows for the Wyndor problem.

	B	C	D	E	F	G	H
16	**Total Profit**			Profit Per Batch for Windows			
17		$36,000	$1,000	$2,000	$3,000	$4,000	$5,000
18		$3,000	$15,000	$18,000	$24,000	$30,000	$36,000
19	Profit Per Batch	$4,000	$19,000	$22,000	$26,000	$32,000	$38,000
20	for Doors	$5,000	$23,000	$26,000	$29,000	$34,000	$40,000
21		$6,000	$27,000	$30,000	$33,000	$36,000	$42,000

However, there is a trick using the & symbol that enables Solver Table to show the results from multiple changing cells and/or output cells within a single cell of the table. We utilize this trick in the Solver Table shown in Fig. 6.18 to show the results for *both* changing cells, DoorBatchesProduced (C12) and WindowBatchesProduced (D12), for each pair of trial values for ProfitPerBatch (C4:D4). The key formula is in cell C25:

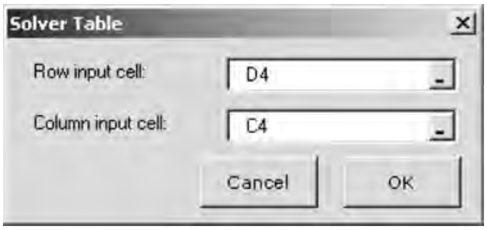
C25 = "("& DoorBatchesProduced &", "& WindowBatchesProduced &")"

The & character tells Excel to concatenate, so the result will be a left parenthesis, followed by the value in DoorBatchesProduced (C12), then a comma and the contents in WindowBatchesProduced (D12), and finally a right parenthesis. If DoorBatchesProduced = 2 and WindowBatchesProduced = 6, the result is (2, 6). Thus, the results from *both* changing cells are displayed within a *single* cell of the table.

After the usual preliminaries in entering the information shown in rows 24–25 and columns B-C of Fig. 6.18, along with the formula in C25, clicking on the OK button

	B	C	D	E	F	G	H
24	Batches Produced (Doors, Windows)			Profit Per Batch for Windows			
25		(2,6)	$1,000	$2,000	$3,000	$4,000	$5,000
26		$3,000	(4,3)	(4,3)	(2,6)	(2,6)	(2,6)
27	Profit Per Batch	$4,000	(4,3)	(4,3)	(2,6)	(2,6)	(2,6)
28	for Doors	$5,000	(4,3)	(4,3)	(4,3)	(2,6)	(2,6)
29		$6,000	(4,3)	(4,3)	(4,3)	(4,3)	(4,3)

	C
25	="(" & DoorBatchesProduced & "," & WindowBatchesProduced & ")"



Range Name	Cells
DoorBatchesProduced	C12
WindowBatchesProduced	D12

■ **FIGURE 6.18**
A two-dimensional application of the Solver Table that shows the effect on the optimal product mix of systematically varying the estimates of the profits per batch of doors and windows for the Wyndor problem.

automatically generates the entire Solver Table. Cells D26:H29 show the optimal solution for the various combinations of trial values for the profits per batch of the doors and windows. The upper right-hand corner (cell H26) of this Solver Table gives the optimal solution of $(x_1, x_2) = (2, 6)$ when using the original profit estimates of \$3,000 per batch of doors and \$5,000 per batch of windows. Moving down from this cell corresponds to increasing this estimate for doors while moving to the left amounts to decreasing the estimate for windows. (The cells when moving up or to the right of H26 are not shown because these changes would only increase the attractiveness of $(x_1, x_2) = (2, 6)$ as the optimal solution.) Note that $(x_1, x_2) = (2, 6)$ continues to be the optimal solution for all the cells near H26. This indicates that the original estimates of profit per batch would need to be very inaccurate indeed before the optimal product mix would change.

Using the Sensitivity Report to Perform Sensitivity Analysis

You now have seen how some sensitivity analysis can be performed readily on a spreadsheet either by interactively making changes in data cells and re-solving or by using the Solver Table to generate similar information systematically. However, there is a shortcut. Some of the same information (and more) can be obtained more quickly and precisely by simply using the sensitivity report provided by the Excel Solver. (Essentially the same sensitivity report is a standard part of the output available from other linear programming software packages as well, including MPL/CPLEX, LINDO, and LINGO.)

Section 4.7 already has discussed the sensitivity report and how it is used to perform sensitivity analysis. Figure 4.10 in that section shows the sensitivity report for the Wyndor problem. Part of this report is shown here in Fig. 6.19. Rather than repeating Sec. 4.7, we will focus here on illustrating how the sensitivity report can efficiently address the specific questions raised in the preceding subsections for the Wyndor problem.

The question considered in the first two subsections was how far the initial estimate of \$3,000 for P_D could be off before the current optimal solution, $(x_1, x_2) = (2, 6)$, would change. Figures 6.10 and 6.11 showed that the optimal solution would not change until

■ **FIGURE 6.19**
Part of the sensitivity report generated by the Excel Solver for the original Wyndor problem (Fig. 6.8), where the last three columns identify the allowable ranges for the profits per batch of doors and windows.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$C\$12	DoorBatchesProduced	2	0	3000	4500	3000
\$D\$12	WindowBatchesProduced	6	0	5000	1E+30	3000

P_D is raised to somewhere between \$5,000 and \$10,000. Figure 6.13 then narrowed down the gap for where the optimal solution changes to somewhere between \$7,000 and \$8,000. This figure also showed that if the initial estimate of \$3,000 for P_D is too high rather than too low, P_D would need to be dropped to somewhere below \$1,000 before the optimal solution would change.

Now look at how the portion of the sensitivity report in Figure 6.19 addresses this same question. The DoorBatchesProduced row in this report provides the following information (without the dollar signs) about P_D.

Current value of P_D:	3,000.	
Allowable increase in P_D:	4,500.	So $P_D \leq 3{,}000 + 4{,}500 = 7{,}500$
Allowable decrease in P_D:	3,000.	So $P_D \geq 3{,}000 - 3{,}000 = 0$.
Allowable range for P_D:		$0 \leq P_D \leq 7{,}500$.

Therefore, if P_D is changed from its current value (without making any other change in the model), the current solution $(x_1, x_2) = (2, 6)$ will remain optimal so long as the new value of P_D is within this *allowable range*, $0 \leq P_D \leq \$7{,}500$.

Figure 6.20 provides graphical insight into this allowable range. For the original value of $P_D = 3{,}000$, the solid line in the figure shows the slope of the objective function line passing through (2, 6). At the lower end of the allowable range, where $P_D = 0$, the objective function line that passes through (2, 6) now is line B in the figure, so every point on the line segment between (0, 6) and (2, 6) is an optimal solution. For any value of $P_D < 0$, the objective function line will have rotated even further so that (0, 6) becomes the only optimal solution. At the upper end of the allowable range, when $P_D = 7{,}500$, the objective function line that passes through (2, 6) becomes line C, so every point on the line segment between (2, 6) and (4, 3) becomes an optimal solution. For any value of $P_D > 7{,}500$, the objective function line is even steeper than line C, so (4, 3) becomes the only optimal solution. Consequently, the original optimal solution, $(x_1, x_2) = (2, 6)$ remains optimal only as long as $0 \leq P_D \leq \$7{,}500$.

The procedure called *Graphical Method and Sensitivity Analysis* in IOR Tutorial is designed to help you perform this kind of graphical analysis. After you enter the model for the original Wyndor problem, the module provides you with the graph shown in Fig. 6.20 (without the dashed lines). You then can simply drag one end of the objective line up or down to see how far you can increase or decrease P_D before $(x_1, x_2) = (2, 6)$ will no longer be optimal.

> *Conclusion:* The allowable range for P_D is $0 \leq P_D \leq \$7{,}500$, because $(x_1, x_2) = (2, 6)$ remains optimal over this range but not beyond. (When $P_D = 0$ or $P_D = \$7{,}500$, there are multiple optimal solutions, but $(x_1, x_2) = (2, 6)$ still is one of them.) With the range this wide around the original estimate of \$3,000

6.8-1. Consider the following problem.

Maximize $\quad Z = 2x_1 + 5x_2,$

subject to

$x_1 + 2x_2 \le 10$ (resource 1)
$x_1 + 3x_2 \le 12$ (resource 2)

and

$x_1 \ge 0, \quad x_2 \ge 0,$

where Z measures the profit in dollars from the two activities.

While doing sensitivity analysis, you learn that the estimates of the unit profits are accurate only to within ±50 percent. In other words, the ranges of *likely values* for these unit profits are \$1 to \$3 for activity 1 and \$2.50 to \$7.50 for activity 2.

E* **(a)** Formulate a spreadsheet model for this problem based on the original estimates of the unit profits. Then use the Solver to find an optimal solution and to generate the sensitivity report.

E* **(b)** Use the spreadsheet and Solver to check whether this optimal solution remains optimal if the unit profit for activity 1 changes from \$2 to \$1. From \$2 to \$3.

E* **(c)** Also check whether the optimal solution remains optimal if the unit profit for activity 1 still is \$2 but the unit profit for activity 2 changes from \$5 to \$2.50. From \$5 to \$7.50.

E* **(d)** Use the Solver Table to systematically generate the optimal solution and total profit as the unit profit of activity 1 increases in 20¢ increments from \$1 to \$3 (without changing the unit profit of activity 2). Then do the same as the unit profit of activity 2 increases in 50¢ increments from \$2.50 to \$7.50 (without changing the unit profit of activity 1). Use these results to estimate the allowable range for the unit profit of each activity.

I **(e)** Use the Graphical Method and Sensitivity Analysis procedure in IOR Tutorial to estimate the allowable range for the unit profit of each activity.

E* **(f)** Use the sensitivity report provided by the Excel Solver to find the allowable range for the unit profit of each activity. Then use these ranges to check your results in parts (*b*–*e*).

E* **(g)** Use a two-way Solver Table to systematically generate the optimal solution as the unit profits of the two activities are changed simultaneously as described in part (*d*).

I **(h)** Use the Graphical Method and Sensitivity Analysis procedure in IOR Tutorial to interpret the results in part (*g*) graphically.

E* **6.8-2.** Reconsider the model given in Prob. 6.8-1. While doing sensitivity analysis, you learn that the estimates of the right-hand sides of the two functional constraints are accurate only to within ±50 percent. In other words, the ranges of *likely values* for these parameters are 5 to 15 for the first right-hand side and 6 to 18 for the second right-hand side.

(a) After solving the original spreadsheet model, determine the shadow price for the first functional constraint by increasing its right-hand side by 1 and solving again.

(b) Use the Solver Table to generate the optimal solution and total profit as the right-hand side of the first functional constraint is incremented by 1 from 5 to 15. Use this table to estimate the allowable range for this right-hand side, i.e., the range over which the shadow price obtained in part (*a*) is valid.

(c) Repeat part (*a*) for the second functional constraint.

(d) Repeat part (*b*) for the second functional constraint where its right-hand side is incremented by 1 from 6 to 18.

(e) Use the Solver's sensitivity report to determine the shadow price for each functional constraint and the allowable range for the right-hand side of each of these constraints.

6.8-3. Consider the following problem.

Maximize $\quad Z = x_1 + 2x_2,$

subject to

$x_1 + 3x_2 \le 8$ (resource 1)
$x_1 + x_2 \le 4$ (resource 2)

and

$x_1 \ge 0, \quad x_2 \ge 0,$

where Z measures the profit in dollars from the two activities and the right-hand sides are the number of units available of the respective resources.

I **(a)** Use the graphical method to solve this model.

I **(b)** Use graphical analysis to determine the shadow price for each of these resources by solving again after increasing the amount of the resource available by 1.

E* **(c)** Use the spreadsheet model and the Solver instead to do parts (*a*) and (*b*).

E* **(d)** For each resource in turn, use the Solver Table to systematically generate the optimal solution and the total profit when the only change is that the amount of that resource available increases in increments of 1 from 4 less than the original value up to 6 more than the current value. Use these results to estimate the allowable range for the amount available of each resource.

(e) Use the Solver's sensitivity report to obtain the shadow prices. Also use this report to find the range for the amount of each resource available over which the corresponding shadow price remains valid.

(f) Describe why these shadow prices are useful when management has the flexibility to change the amounts of the resources being made available.

6.8-4.* One of the products of the G.A. Tanner Company is a special kind of toy that provides an estimated unit profit of \$3. Because of a large demand for this toy, management would like to increase its production rate from the current level of 1,000 per day.

However, a limited supply of two subassemblies (A and B) from vendors makes this difficult. Each toy requires two subassemblies of type A, but the vendor providing these subassemblies would only be able to increase its supply rate from the current 2,000 per day to a maximum of 3,000 per day. Each toy requires only one subassembly of type B, but the vendor providing these subassemblies would be unable to increase its supply rate above the current level of 1,000 per day. Because no other vendors currently are available to provide these subassemblies, management is considering initiating a new production process internally that would simultaneously produce an equal number of subassemblies of the two types to supplement the supply from the two vendors. It is estimated that the company's cost for producing one subassembly of each type would be $2.50 more than the cost of purchasing these subassemblies from the two vendors. Management wants to determine both the production rate of the toy and the production rate of each pair of subassemblies (one A and one B) that would maximize the total profit.

The following table summarizes the data for the problem.

	Resource Usage per Unit of Each Activity		
	Activity		
Resource	Produce Toys	Produce Subassemblies	Amount of Resource Available
Subassembly A	2	–1	3,000
Subassembly B	1	–1	1,000
Unit profit	$3	–$2.50	

E* **(a)** Formulate and solve a spreadsheet model for this problem.

E* **(b)** Since the stated unit profits for the two activities are only estimates, management wants to know how much each of these estimates can be off before the optimal solution would change. Begin exploring this question for the first activity (producing toys) by using the spreadsheet and Solver to manually generate a table that gives the optimal solution and total profit as the unit profit for this activity increases in 50¢ increments from $2 to $4. What conclusion can be drawn about how much the estimate of this unit profit can differ in each direction from its original value of $3 before the optimal solution would change?

E* **(c)** Repeat part (*b*) for the second activity (producing subassemblies) by generating a table as the unit profit for this activity increases in 50¢ increments from –$3.50 to –$1.50 (with the unit profit for the first activity fixed at $3).

E* **(d)** Use the Solver Table to systematically generate all the data requested in parts (*b*) and (*c*), except use 25¢ increments instead of 50¢ increments. Use these data to refine your conclusions in parts (*b*) and (*c*).

I **(e)** Use the Graphical Method and Sensitivity Analysis procedure in IOR Tutorial to determine how much the unit profit of each activity can change in either direction (without changing the unit profit of the other activity) before the optimal solution would change. Use this information to specify the allowable range for the unit profit of each activity.

E* **(f)** Use Excel's sensitivity report to find the allowable range for the unit profit of each activity.

E* **(g)** Use a two-way Solver Table to systematically generate the optimal solution as the unit profits of the two activities are changed simultaneously as described in parts (*b*) and (*c*).

(h) Use the information provided by Excel's sensitivity report to describe how far the unit profits of the two activities can change simultaneously before the optimal solution might change.

E* **6.8-5.** Reconsider Prob. 6.8-4. After further negotiations with each vendor, management of the G.A. Tanner Co. has learned that either of them would be willing to consider increasing their supply of their respective subassemblies over the previously stated maxima (3,000 subassemblies of type A per day and 1,000 of type B per day) if the company would pay a small premium over the regular price for the extra subassemblies. The size of the premium for each type of subassembly remains to be negotiated. The demand for the toy being produced is sufficiently high so that 2,500 per day could be sold if the supply of subassemblies could be increased enough to support this production rate. Assume that the original estimates of unit profits given in Prob. 6.8-4 are accurate.

(a) Formulate and solve a spreadsheet model for this problem with the original maximum supply levels and the additional constraint that no more than 2,500 toys should be produced per day.

(b) Without considering the premium, use the spreadsheet and Solver to determine the shadow price for the subassembly A constraint by solving the model again after increasing the maximum supply by 1. Use this shadow price to determine the maximum premium that the company should be willing to pay for each subassembly of this type.

(c) Repeat part (*b*) for the subassembly B constraint.

(d) Estimate how much the maximum supply of subassemblies of type A could be increased before the shadow price (and the corresponding premium) found in part (*b*) would no longer be valid by using the Solver Table to generate the optimal solution and total profit (excluding the premium) as the maximum supply increases in increments of 100 from 3,000 to 4,000.

(e) Repeat part (*d*) for subassemblies of type B by using the Solver Table as the maximum supply increases in increments of 100 from 1,000 to 2,000.

(f) Use the Solver's sensitivity report to determine the shadow price for each of the subassembly constraints and the allowable range for the right-hand side of each of these constraints.

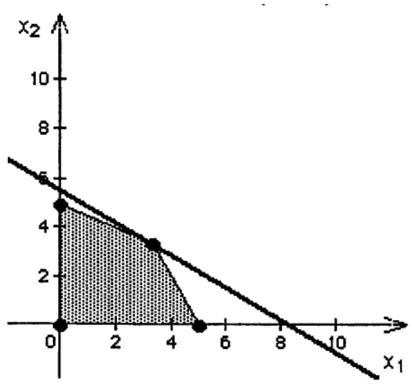
Solutions to Problems 3.5.2--3.5.5 and 6.8.1--6.8.5, of Hillier and Lieberman's book:

Solution: 3.5-2.

(a) maximize $P = 20x_1 + 30x_2$

subject to
$$\begin{aligned} 2x_1 + \quad x_2 &\leq 10 \\ 3x_1 + \ 3x_2 &\leq 20 \\ 2x_1 + \ 4x_2 &\leq 20 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(b) Optimal Solution: $(x_1^*, x_2^*) = \left(3\frac{1}{3}, 3\frac{1}{3}\right)$ and $P^* = 166.67$



(c) - (e)

Resource	Resource Usage Per Unit of Each Activity Activity 1	Activity 2	Totals		Resource Available
1	2	1	10	≤	10
2	3	3	20	≤	20
3	2	4	20	≤	20
Unit Profit	20	30	**$ 166.67**		
Solution	**3.333**	**3.333**			

(d)

(x_1, x_2)	Feasible?	P
$(2, 2)$	Yes	\$100
$(3, 3)$	Yes	\$150
$(2, 4)$	Yes	\$160 Best
$(4, 2)$	Yes	\$140
$(3, 4)$	No	
$(4, 3)$	No	

Solution: 3.5-3.

(a) maximize $P = 300A + 250B + 200C$

subject to
$$0.02A + 0.03B + 0.05C \leq 40$$
$$0.05A + 0.02B + 0.04C \leq 40$$

and $A, B, C \geq 0$

(b)

	Resource Usage Per Unit of Each Activity					Resource
Resource	Part A	Part B	Part C	Total		Available
Machine 1	0.02	0.03	0.05		≤	40
Machine 2	0.05	0.02	0.04		≤	40
Unit Profit	\$300	\$250	\$200			
Solution						

(c) Many answers are possible.

(A, B, C)	Feasible?	P
$(500, 500, 300)$	No	
$(350, 1000, 0)$	Yes	\$355,000
$(400, 1000, 0)$	Yes	\$370,000 Best

(d)

	Resource Usage Per Unit of Each Activity					Resource
Resource	Part A	Part B	Part C	Total		Available
Machine 1	0.02	0.03	0.05	40	≤	40
Machine 2	0.05	0.02	0.04	40	≤	40
Unit Profit	\$300	\$250	\$200	381818		
Solution	363.6363636	1090.909091	0			

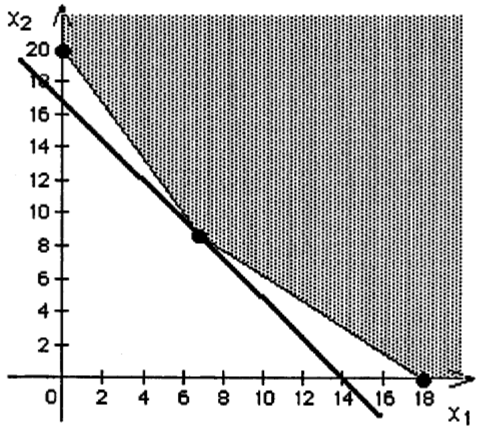
Solution: 3.5-4.

(a) minimize $C = 60x_1 + 50x_2$

subject to
$$5x_1 + 3x_2 \geq 60$$
$$2x_1 + 2x_2 \geq 30$$
$$7x_1 + 9x_2 \geq 126$$

and $x_1, x_2 \geq 0$

(b) Optimal Solution: $(x_1^*, x_2^*) = (6.75, 8.75)$ and $C^* = 842.50$



(c) - (e)

Benefit	Benefit Contribution Per Unit of Each Activity Activity 1	Activity 2	Totals		Minimum Level
1	5	3	60	≥	60
2	2	2	31	≥	30
3	7	9	126	≥	126
Unit Cost	60	50	**$ 842.50**		
Solution	**6.75**	**8.75**			

(d)

(x_1, x_2)	Feasible?	C
$(7, 7)$	No	
$(7, 8)$	No	
$(8, 7)$	No	
$(8, 8)$	Yes	$880 Best
$(8, 9)$	Yes	$930
$(9, 8)$	Yes	$940

Solution: 3.5-5.

(a) minimize $C = 84C + 72T + 60A$

subject to

$$90C + 20T + 40A \geq 200$$
$$30C + 80T + 60A \geq 180$$
$$10C + 20T + 60A \geq 150$$

and $C, T, A \geq 0$

(b) - (e)

Nutritional Ingredient	Corn	Kilogram of Tankage	Alfalfa	Totals		Minimum Level
Carbohydrates	90	20	40	200	≥	200
Proteins	30	80	60	180	≥	180
Vitamins	10	20	60	157	≥	150
Unit Cost	84	72	60	**$ 242**		
Solution	**1**	**0**	**2**			

(c) $(x_1, x_2, x_3) = (1, 2, 2)$ is a feasible solution with a daily cost of \$348. This diet will provide 210 kg of carbohydrates, 310 kg of protein, and 170 kg of vitamins daily.

(d) Answers will vary.

Solution: 6.8-1.

(a)

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$2	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	10	2	10
6	Resource 2	1	3	12	2	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	6	2			$22

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution Activity 1	6	0	2	0.5	0.33333
$C$9	Solution Activity 2	2	0	5	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Resource 1 Used	10	1	10	2	2
$D$6	Resource 2 Used	12	1	12	3	2

(b) The optimal solution is $(0, 4)$ if the unit profit for Activity 1 is $1.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$1	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	8	2	10
6	Resource 2	1	3	12	2	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	0	4			$20

The optimal solution is $(10, 0)$ if the unit profit for Activity 1 is $3.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$3	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	10	2	10
6	Resource 2	1	3	10	2	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	10	0			$30

(c) The optimal solution is $(10, 0)$ if the unit profit for Activity 2 is \$2.50.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	\$2	\$2.50			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	10	≤	10
6	Resource 2	1	3	10	≤	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	10	0			\$20

The optimal solution is $(0, 4)$ if the unit profit for Activity 2 is \$7.50.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	\$2	\$7.50			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	8	≤	10
6	Resource 2	1	3	12	≤	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	0	4			\$30

(d)

	A	B	C	D
11	Unit Profit for	Solution		Total
12	Activity 1	Activity 1	Activity 2	Profit
13		6	2	\$22.00
14	\$1.00	0	4	\$20.00
15	\$1.20	0	4	\$20.00
16	\$1.40	0	4	\$20.00
17	\$1.60	0	4	\$20.00
18	\$1.80	6	2	\$20.80
19	\$2.00	6	2	\$22.00
20	\$2.20	6	2	\$23.20
21	\$2.40	6	2	\$24.40
22	\$2.60	10	0	\$26.00
23	\$2.80	10	0	\$28.00
24	\$3.00	10	0	\$30.00

27	Unit Profit for	Solution		Total
28	Activity 2	Activity 1	Activity 2	Profit
29		6	2	$22.00
30	$2.50	10	0	$20.00
31	$3.00	10	0	$20.00
32	$3.50	10	0	$20.00
33	$4.00	6	2	$20.00
34	$4.50	6	2	$21.00
35	$5.00	6	2	$22.00
36	$5.50	6	2	$23.00
37	$6.00	0	4	$24.00
38	$6.50	0	4	$26.00
39	$7.00	0	4	$28.00
40	$7.50	0	4	$30.00

The allowable range for the unit profit of Activity 1 is approximately between $1.60 and $1.80 up to between $2.40 and $2.60. The allowable range for the unit profit of Activity 2 is between $3.50 and $4 up to between $5.50 and $6.

(e) The allowable range for the unit profit of Activity 1 is approximately between $1.67 and $2.50. The allowable range for the unit profit of Activity 2 is between $4 and $6.

Objective Coefficient		
Current Value	Allowable Range to Stay Optimal	
	Minimum	Maximum
2	1.67	2.5
5	4	6

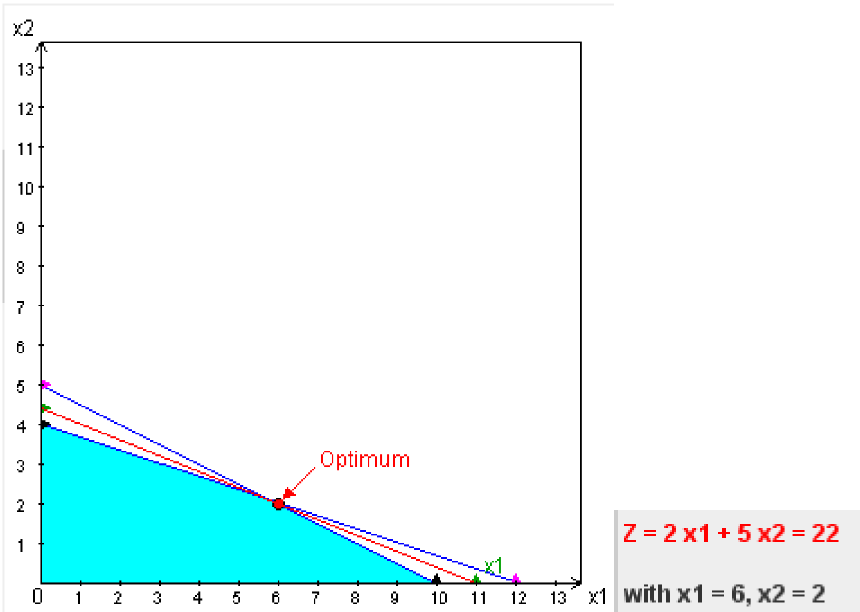
(f) The allowable range for the unit profit of Activity 1 is approximately between $1.67 and $2.50. The allowable range for the unit profit of Activity 2 is between $4 and $6.

(g)

	A	B	C	D	E	F	G	H	I	J	K	L	M
11	Total Profit						Unit Profit for Activity 2						
12		$22	$2.50	$3.00	$3.50	$4.00	$4.50	$5.00	$5.50	$6.00	$6.50	$7.00	$7.50
13		$1.00	$11.00	$12.00	$14.00	$16.00	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
14		$1.20	$12.20	$13.20	$14.20	$16.00	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
15		$1.40	$14.00	$14.40	$15.40	$16.40	$18.00	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
16	Unit Profit	$1.60	$16.00	$16.00	$16.60	$17.60	$18.60	$20.00	$22.00	$24.00	$26.00	$28.00	$30.00
17	for	$1.80	$18.00	$18.00	$18.00	$18.80	$19.80	$20.80	$22.00	$24.00	$26.00	$28.00	$30.00
18	Activity 1	$2.00	$20.00	$20.00	$20.00	$20.00	$21.00	$22.00	$23.00	$24.00	$26.00	$28.00	$30.00
19		$2.20	$22.00	$22.00	$22.00	$22.00	$22.20	$23.20	$24.20	$25.20	$26.20	$28.00	$30.00
20		$2.40	$24.00	$24.00	$24.00	$24.00	$24.00	$24.40	$25.40	$26.40	$27.40	$28.40	$30.00
21		$2.60	$26.00	$26.00	$26.00	$26.00	$26.00	$26.00	$26.60	$27.60	$28.60	$29.60	$30.60
22		$2.80	$28.00	$28.00	$28.00	$28.00	$28.00	$28.00	$28.00	$28.80	$29.80	$30.80	$31.80
23		$3.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$30.00	$31.00	$32.00	$33.00
24													

25	Solution						Unit Profit for Activity 2						
26		(6,2)	$2.50	$3.00	$3.50	$4.00	$4.50	$5.00	$5.50	$6.00	$6.50	$7.00	$7.50
27		$1.00	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)
28		$1.20	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)
29		$1.40	(10,0)	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)
30	Unit Profit	$1.60	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)
31	for	$1.80	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)	(0,4)
32	Activity 1	$2.00	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)	(0,4)	(0,4)
33		$2.20	(10,0)	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)	(6,2)	(0,4)	(0,4)
34		$2.40	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)	(6,2)	(0,4)
35		$2.60	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)	(6,2)
36		$2.80	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)
37		$3.00	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(10,0)	(6,2)	(6,2)	(6,2)	(6,2)

(h) Keeping the unit profit of Activity 2 fixed, the unit profit of Activity 1 cannot be changed to less than 1.67 or more than 2.5 without changing the optimal solution. Similarly if the unit profit of Activity 1 is fixed at 1, the unit profit of Activity 2 needs to stay between 4 and 6 so that the optimal solution remains the same. Otherwise, the objective function line becomes either too flat or too steep and the optimal solution becomes $(0, 4)$ or $(10, 0)$.



Solution: 6.8-2.

(a) The original model:

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$2	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	10	2	10
6	Resource 2	1	3	12	2	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	6	2			$22.00

With one additional unit of resource 1:

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$2	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	11	2	11
6	Resource 2	1	3	12	2	12
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	9	1			$23.00

The shadow price (the increase in total profit) is $1.

(b) The shadow price of $1 is valid in the range of 8 to 12.

	A	B	C	D	E
12	Available	Solution		Total	Incremental
13	Resource 1	Activity 1	Activity 2	Profit	Profit
14		6	2	$22.00	
15	5	0	2.5	$12.50	
16	6	0	3	$15.00	$2.50
17	7	0	3.5	$17.50	$2.50
18	8	0	4	$20.00	$2.50
19	9	3	3	$21.00	$1.00
20	10	6	2	$22.00	$1.00
21	11	9	1	$23.00	$1.00
22	12	12	0	$24.00	$1.00
23	13	12	0	$24.00	$0.00
24	14	12	0	$24.00	$0.00
25	15	12	0	$24.00	$0.00

(c) With one additional unit of resource 2:

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$2	$5			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	2	10	≤	10
6	Resource 2	1	3	13	≤	13
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	4	3			$23.00

The shadow price (the increase in total profit) is $1.

(d) The shadow price of $1 is valid in the range of 10 to 15.

	A	B	C	D	E
12	Available	Solution		Total	Incremental
13	Resource 2	Activity 1	Activity 2	Profit	Profit
14		6	2	$22.00	
15	6	6	0	$12.00	
16	7	7	0	$14.00	$2.00
17	8	8	0	$16.00	$2.00
18	9	9	0	$18.00	$2.00
19	10	10	0	$20.00	$2.00
20	11	8	1	$21.00	$1.00
21	12	6	2	$22.00	$1.00
22	13	4	3	$23.00	$1.00
23	14	2	4	$24.00	$1.00
24	15	0	5	$25.00	$1.00
25	16	0	5	$25.00	$0.00
26	17	0	5	$25.00	$0.00
27	18	0	5	$25.00	$0.00

(e) From the sensitivity report, the shadow prices for both constraints are \$1. According to the allowable increase and decrease, the allowable range for the right-hand side of the first constraint is 8 to 12. Similarly, the allowable range for the right-hand side of the second constraint is 10 to 15.

Adjustable Cells

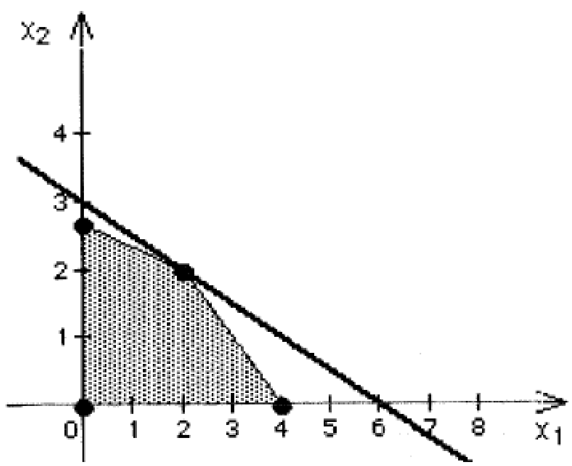
Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$9	Solution Activity 1	6	0	2	0.5	0.333
\$C\$9	Solution Activity 2	2	0	5	1	1

Constraints

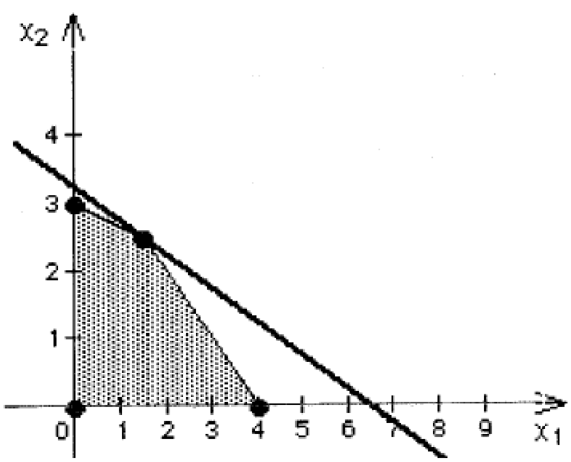
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$5	Resource 1 Used	10	1	10	2	2
\$D\$6	Resource 2 Used	12	1	12	3	2

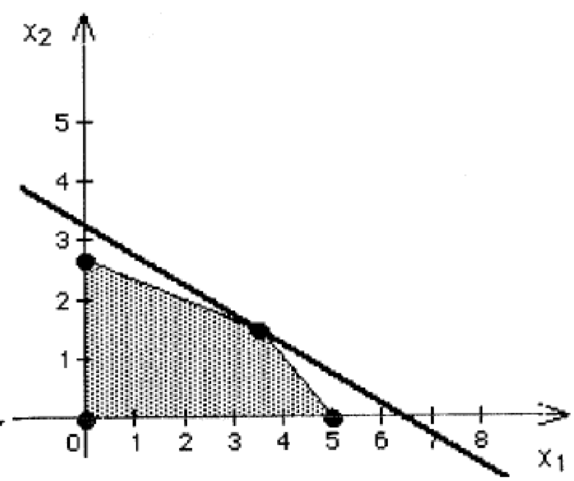
Solution: 6.8-3.

(a) Optimal Solution: $(x_1, x_2) = (2, 2)$, with profit \$6



(b)





(c) The original model:

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$1	$2			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	3	8	≤	8
6	Resource 2	1	1	4	≤	4
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	2	2			$6.00

The shadow price for resource 1 is $0.50.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$1	$2			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	3	9	≤	9
6	Resource 2	1	1	4	≤	4
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	1.5	2.5			$6.50

The shadow price for resource 2 is $0.50.

	A	B	C	D	E	F
1		Activity 1	Activity 2			
2	Unit Profit	$1	$2			
3						
4		Resource Usage		Used		Available
5	Resource 1	1	3	8	≤	8
6	Resource 2	1	1	5	≤	5
7						
8		Activity 1	Activity 2			Total Profit
9	Solution	3.5	1.5			$6.50

(d) The allowable range for the right-hand side of the resource 1 constraint is approximately between 4 (or less) and 12.

	A	B	C	D	E
12	Available	Solution		Total	Incremental
13	Resource 1	Activity 1	Activity 2	Profit	Profit
14		2	2	$6.00	
15	4	4	0	$4.00	
16	5	3.5	0.5	$4.50	$0.50
17	6	3	1	$5.00	$0.50
18	7	2.5	1.5	$5.50	$0.50
19	8	2	2	$6.00	$0.50
20	9	1.5	2.5	$6.50	$0.50
21	10	1	3	$7.00	$0.50
22	11	0.5	3.5	$7.50	$0.50
23	12	0	4	$8.00	$0.50
24	13	0	4	$8.00	$0.00
25	14	0	4	$8.00	$0.00

The allowable range for the right-hand side of the resource 2 constraint is approximately between 3 and 8.

	A	B	C	D	E
28	Available	Solution		Total	Incremental
29	Resource 2	Activity 1	Activity 2	Profit	Profit
30		2	2	$6.00	
31	0	0	0	$0.00	
32	1	0	1	$2.00	$2.00
33	2	0	2	$4.00	$2.00
34	3	0.5	2.5	$5.50	$1.50
35	4	2	2	$6.00	$0.50
36	5	3.5	1.5	$6.50	$0.50
37	6	5	1	$7.00	$0.50
38	7	6.5	0.5	$7.50	$0.50
39	8	8	0	$8.00	$0.50
40	9	8	0	$8.00	$0.00
41	10	8	0	$8.00	$0.00

(e) The shadow price for both resources is $0.50. The allowable range for the right-hand side of the first resource is between 4 and 12 and that of the second resource is between 2.667 and 8.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Solution Activity 1	2	0	1	1	0.333
$C$9	Solution Activity 2	2	0	2	1	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Resource 1 Used	8	0.5	8	4	4
$D$6	Resource 2 Used	4	0.5	4	4	1.333

(f) These shadow prices tell management that for each additional unit of resource, the profit increases by $0.50 (for small changes). Management is then able to evaluate whether or not to change the available amount of resources.

Solution: 6.8-4.

(a)

	A	B	C	D	E	F
1		Toys	Subassemblies			
2	Unit Profit	$3.00	-$2.50			
3						
4		Resource Usage		Used		Available
5	Subassembly A	2	-1	3,000	≥	3,000
6	Subassembly B	1	-1	1,000	≥	1,000
7						
8		Toys	Subassemblies			Total Profit
9	Production	2,000	1,000			$3,500

(b)

Unit Profit for Toys	Optimal Production Rates		Total Profit
	Toys	Subassemblie s	
$2.00	1000	0	$2000
$2.50	1000	0	$2500
$3.00	2000	1000	$3500
$3.50	2000	1000	$4500
$4.00	2000	1000	$5500

The estimate of the unit profit for toys can be off by something between 0 and 0.50 before the optimal solution changes. There is no change in the solution for an increase in the unit profit for toys, at least for an increase up to $1.

(c)

Unit Profit for Subassemblies	Optimal Production Rates		Total Profit
	Toys	Subassemblies	
-$3.50	1000	0	$3000
-$3.00	1000	0	$3000
-$2.50	2000	1000	$3500
-$2.00	2000	1000	$4000
-$1.50	2000	1000	$4500

The estimate of the unit profit for subassemblies can be off by something between 0 and 0.50 before the optimal solution changes. There is no change in the solution for an increase in the unit profit for subassemblies, at least for an increase up to $1.

(d) Solver Table for change in unit profit for toys as in (b):

	A	B	C	D
11	Unit Profit	Production		
12	for Toys	Toys	Subassemblies	Total Profit
13		2,000	1,000	$3,500
14	$2.00	1000	0	$2,000
15	$2.25	1000	0	$2,250
16	$2.50	1000	0	$2,500
17	$2.75	2000	1000	$3,000
18	$3.00	2000	1000	$3,500
19	$3.25	2000	1000	$4,000
20	$3.50	2000	1000	$4,500
21	$3.75	2000	1000	$5,000
22	$4.00	2000	1000	$5,500

Solver Table for change in unit profit for subassemblies as in (c):

	A	B	C	D
11	Unit Profit	Production		
12	for Subassemblies	Toys	Subassemblies	Total Profit
13		2,000	1,000	$3,500
14	-$3.50	1000	0	$3,000
15	-$3.25	1000	0	$3,000
16	-$3.00	1000	0	$3,000
17	-$2.75	2000	1000	$3,250
18	-$2.50	2000	1000	$3,500
19	-$2.25	2000	1000	$3,750
20	-$2.00	2000	1000	$4,000
21	-$1.75	2000	1000	$4,250
22	-$1.50	2000	1000	$4,500

(e) The unit profit for toys can vary between $2.50 and $5 before the solution changes. For subassemblies, the unit profit can change between -$3 and -1.50 before the solution changes.

(f) The allowable range of the unit profit for toys is $2.50 to $5 whereas that for subassemblies is -$3 to -$1.50.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Production Toys	2,000	0	3	2	0.5
$C$9	Production Subassemblies	1,000	0	-2.5	1	0.5

(g)

	A	B	C	D	E	F	G	H	I	J	K
11	Total Profit					Unit Profit for Subassemblies					
12		$3,500	-$3.50	-$3.25	-$3.00	-$2.75	-$2.50	-$2.25	-$2.00	-$1.75	-$1.50
13		$2.00	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,000	$2,250	$2,500
14		$2.25	$2,250	$2,250	$2,250	$2,250	$2,250	$2,250	$2,500	$2,750	$3,000
15		$2.50	$2,500	$2,500	$2,500	$2,500	$2,500	$2,750	$3,000	$3,250	$3,500
16	Unit Profit	$2.75	$2,750	$2,750	$2,750	$2,750	$3,000	$3,250	$3,500	$3,750	$4,000
17	for Toys	$3.00	$3,000	$3,000	$3,000	$3,250	$3,500	$3,750	$4,000	$4,250	$4,500
18		$3.25	$3,250	$3,250	$3,500	$3,750	$4,000	$4,250	$4,500	$4,750	$5,000
19		$3.50	$3,500	$3,750	$4,000	$4,250	$4,500	$4,750	$5,000	$5,250	$5,500
20		$3.75	$4,000	$4,250	$4,500	$4,750	$5,000	$5,250	$5,500	$5,750	$6,000
21		$4.00	$4,500	$4,750	$5,000	$5,250	$5,500	$5,750	$6,000	$6,250	$6,500

(h) As long as the sum of the percentage change of the unit profit for subassemblies does not exceed 100% (where the allowable range is given in part (f)), the solution does not change.

Solution: 6.8-5.

(a)

	A	B	C	D	E	F
1		Toys	Subassemblies			
2	Unit Profit	$3.00	-$2.50			
3						
4		Resource Usage		Used		Available
5	Subassembly A	2	-1	3,000	≤	3,000
6	Subassembly B	1	-1	1,000	≤	1,000
7						
8		Toys	Subassemblies			Total Profit
9	Production	2,000	1,000			$3,500.00
10		≤				
11		2,500				

(b)

	A	B	C	D	E	F
1		Toys	Subassemblies			
2	Unit Profit	$3.00	-$2.50			
3						
4		Resource Usage		Used		Available
5	Subassembly A	2	-1	3,001	≤	3,001
6	Subassembly B	1	-1	1,000	≤	1,000
7						
8		Toys	Subassemblies			Total Profit
9	Production	2,001	1,001			$3,500.50
10		≤				
11		2,500				

The shadow price for subassembly A is $0.50, which is the maximum premium that the company should be willing to pay.

(c)

	A	B	C	D	E	F
1		Toys	Subassemblies			
2	Unit Profit	$3.00	-$2.50			
3						
4		Resource Usage		Used		Available
5	Subassembly A	2	-1	3,000	≤	3,000
6	Subassembly B	1	-1	1,001	≤	1,001
7						
8		Toys	Subassemblies			Total Profit
9	Production	1,999	998			$3,502.00
10		≤				
11		2,500				

The shadow price for subassembly B is $2, which is the maximum premium that the company should be willing to pay.

(d)

	A	B	C	D	E
14	Available	Production		Total	Incremental
15	Subassembly A	Toys	Subassemblies	Profit	Profit
16		2,000	1,000	$3,500.00	
17	3,000	2,000	1,000	$3,500.00	
18	3,100	2,100	1,100	$3,550.00	$50.00
19	3,200	2,200	1,200	$3,600.00	$50.00
20	3,300	2,300	1,300	$3,650.00	$50.00
21	3,400	2,400	1,400	$3,700.00	$50.00
22	3,500	2,500	1,500	$3,750.00	$50.00
23	3,600	2,500	1,500	$3,750.00	$0.00
24	3,700	2,500	1,500	$3,750.00	$0.00
25	3,800	2,500	1,500	$3,750.00	$0.00
26	3,900	2,500	1,500	$3,750.00	$0.00
27	4,000	2,500	1,500	$3,750.00	$0.00

The shadow price is still valid until the maximum supply of subassembly A is at least 3,500.

(e)

	A	B	C	D	E
14	Available	Production		Total	Incremental
15	Subassembly B	Toys	Subassemblies	Profit	Profit
16		2,000	1,000	$3,500.00	
17	1,000	2,000	1,000	$3,500.00	
18	1,100	1,900	800	$3,700.00	$200.00
19	1,200	1,800	600	$3,900.00	$200.00
20	1,300	1,700	400	$4,100.00	$200.00
21	1,400	1,600	200	$4,300.00	$200.00
22	1,500	1,500	0	$4,500.00	$200.00
23	1,600	1,500	0	$4,500.00	$0.00
24	1,700	1,500	0	$4,500.00	$0.00
25	1,800	1,500	0	$4,500.00	$0.00
26	1,900	1,500	0	$4,500.00	$0.00
27	2,000	1,500	0	$4,500.00	$0.00

The shadow price is still valid until the maximum supply of subassembly A is at least 1,500.

(f)

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$9	Production Toys	2,000	0	3	2	0.5
$C$9	Production Subassemblies	1,000	0	-2.5	1	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$5	Subassembly A Used	3,000	0.5	3000	500	1000
$D$6	Subassembly B Used	1,000	2	1000	500	500

As shown in the sensitivity report, the shadow price is $0.50 for subassembly A and $2 for subassembly B. According to the allowable increase and decrease, the allowable range for the right-hand side of the subassembly A constraint is 2,000 to 3,500. The allowable range for the right-hand side of the subassembly B constraint is 500 to 1,500.

CASES

CASE 8.1 Shipping Wood to Market

Alabama Atlantic is a lumber company that has three sources of wood and five markets to be supplied. The annual availability of wood at sources 1, 2, and 3 is 15, 20, and 15 million board feet, respectively. The amount that can be sold annually at markets 1, 2, 3, 4, and 5 is 11, 12, 9, 10, and 8 million board feet, respectively.

In the past the company has shipped the wood by train. However, because shipping costs have been increasing, the alternative of using ships to make some of the deliveries is being investigated. This alternative would require the company to invest in some ships. Except for these investment costs, the shipping costs in thousands of dollars per million board feet by rail and by water (when feasible) would be the following for each route:

	Unit Cost by Rail ($1,000's) Market					Unit Cost by Ship ($1,000's) Market				
Source	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	61	72	45	55	66	31	38	24	—	35
2	69	78	60	49	56	36	43	28	24	31
3	59	66	63	61	47	—	33	36	32	26

The capital investment (in thousands of dollars) in ships required for each million board feet to be transported annually by ship along each route is given as follows:

	Investment for Ships ($1,000's) Market				
Source	**1**	**2**	**3**	**4**	**5**
1	275	303	238	—	285
2	293	318	270	250	265
3	—	283	275	268	240

Considering the expected useful life of the ships and the time value of money, the equivalent uniform annual cost of these investments is one-tenth the amount given in the table. The objective is to determine the overall shipping plan that minimizes the total equivalent uniform annual cost (including shipping costs).

You are the head of the OR team that has been assigned the task of determining this shipping plan for each of the following three options.

Option 1: Continue shipping exclusively by rail.
Option 2: Switch to shipping exclusively by water (except where only rail is feasible).
Option 3: Ship by either rail or water, depending on which is less expensive for the particular route.

Present your results for each option. Compare.

Finally, consider the fact that these results are based on *current* shipping and investment costs, so the decision on the option to adopt now should take into account management's projection of how these costs are likely to change in the future. For each option, describe a scenario of future cost changes that would justify adopting that option now.

(*Note:* Data files for this case are provided on the book's website for your convenience.)

CASE 8.1 SHIPPING WOOD TO MARKET

Alabama Atlantic is a lumber company that has three sources of wood and five markets to be supplied. The annual availability of wood at sources 1, 2, and 3 is 15, 20, and 15 million board feet, respectively. The amount that can be sold annually at markets 1, 2, 3, 4, and 5 is 11, 12, 9, 10, and 8 million board feet, respectively.

In the past the company has shipped the wood by train. However, because shipping costs have been increasing, the alternative of using ships to make some of the deliveries is being investigated. This alternative would require the company to invest in some ships. Except for these investment costs, the shipping costs in thousands of dollars per million board feet by rail and by water (when feasible) would be the following for each route:

	Unit Cost by Rail ($1,000's) Market					Unit Cost by Ship ($1,000's) Market				
Source	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1	61	72	45	55	66	31	38	24	—	35
2	69	78	60	49	56	36	43	28	24	31
3	59	66	63	61	47	—	33	36	32	26

The capital investment (in thousands of dollars) in ships required for each million board feet to be transported annually by ship along each route is given as follows:

	Investment for Ships ($1,000's) Market				
Source	**1**	**2**	**3**	**4**	**5**
1	275	303	238	—	285
2	293	318	270	250	265
3	—	283	275	268	240

Considering the expected useful life of the ships and the time value of money, the equivalent uniform annual cost of these investments is one-tenth the amount given in the table. The objective is to determine the overall shipping plan that minimizes the total equivalent uniform annual cost (including shipping costs).

You are the head of the OR team that has been assigned the task of determining this shipping plan for each of the following three options.

Option 1: Continue shipping exclusively by rail.
Option 2: Switch to shipping exclusively by water (except where only rail is feasible).
Option 3: Ship by either rail or water, depending on which is less expensive for the particular route.

Present your results for each option. Compare.

Finally, consider the fact that these results are based on *current* shipping and investment costs, so that the decision on the option to adopt now should take into account management's projection of how these costs are likely to change in the future. For each option, describe a scenario of future cost changes that would justify adopting that option now.

CASE 8.3 PROJECT PICKINGS

Tazer, a pharmaceutical manufacturing company, entered the pharmaceutical market 12 years ago with the introduction of six new drugs. Five of the six drugs were simply permutations of existing drugs and therefore did not sell very heavily. The sixth drug, however, addressed hypertension and was a huge success. Since Tazer had a patent on the hypertension drug, it experienced no competition, and profits from the hypertension drug alone kept Tazer in business.

During the past 12 years, Tazer continued a moderate amount of research and development, but it never stumbled upon a drug as successful as the hypertension drug. One reason is that the company never had the motivation to invest heavily in innovative research and development. The company was riding the profit wave generated by its hypertension drug and did not feel the need to commit significant resources to finding new drug breakthroughs.

Now Tazer is beginning to fear the pressure of competition. The patent for the hypertension drug expires in 5 years,[1] and Tazer knows that once the patent expires, generic drug manufacturing companies will swarm into the market like vultures. Historical trends show that generic drugs decreased sales of branded drugs by 75 percent.

Tazer is therefore looking to invest significant amounts of money in research and development this year to begin the search for a new breakthrough drug that will offer the company the same success as the hypertension drug. Tazer believes that if the company begins extensive research and development now, the probability of finding a successful drug shortly after the expiration of the hypertension patent will be high.

As head of research and development at Tazer, you are responsible for choosing potential projects and assigning project directors to lead each of the projects. After researching the needs of the market, analyzing the shortcomings of current drugs, and interviewing numerous scientists concerning the promising areas of medical research, you have decided that your department will pursue five separate projects, which are listed below:

Project Up	Develop an antidepressant that does not cause serious mood swings.
Project Stable	Develop a drug that addresses manic-depression.
Project Choice	Develop a less intrusive birth control method for women.
Project Hope	Develop a vaccine to prevent HIV infection.
Project Release	Develop a more effective drug to lower blood pressure.

[1]In general, patents protect inventions for 17 years. In 1995, GATT legislation extending the protection given by new pharmaceutical patents to 20 years became effective. The patent for Tazer's hypertension drug was issued prior to the GATT legislation, however. Thus, the patent only protects the drug for 17 years.

For each of the five projects, you are only able to specify the medical ailment the research should address, since you do not know what compounds will exist and be effective without research.

You also have five senior scientists to lead the five projects. You know that scientists are very temperamental people and will work well only if they are challenged and motivated by the project. To ensure that the senior scientists are assigned to projects they find motivating, you have established a bidding system for the projects. You have given each of the five scientists 1000 bid points. They assign bids to each project, giving a higher number of bid points to projects they most prefer to lead. The following table provides the bids from the five individual senior scientists for the five individual projects:

Project	Dr. Kvaal	Dr. Zuner	Dr. Tsai	Dr. Mickey	Dr. Rollins
Project Up	100	0	100	267	100
Project Stable	400	200	100	153	33
Project Choice	200	800	100	99	33
Project Hope	200	0	100	451	34
Project Release	100	0	600	30	800

You decide to evaluate a variety of scenarios you think are likely.

(a) Given the bids, you need to assign one senior scientist to each of the five projects to maximize the preferences of the scientists. What are the assignments?

(b) Dr. Rollins is being courted by Harvard Medical School to accept a teaching position. You are fighting desperately to keep her at Tazer, but the prestige of Harvard may lure her away. If this were to happen, the company would give up the project with the least enthusiasm. Which project would not be done?

(c) You do not want to sacrifice any project, since researching only four projects decreases the probability of finding a breakthrough new drug. You decide that either Dr. Zuner or Dr. Mickey could lead two projects. Under these new conditions with just four senior scientists, which scientists will lead which projects to maximize preferences?

(d) After Dr. Zuner was informed that she and Dr. Mickey are being considered for two projects, she decided to change her bids. The following table shows Dr. Zuner's new bids for each of the projects:

Project Up	20
Project Stable	450
Project Choice	451
Project Hope	39
Project Release	40

Under these new conditions with just four scientists, which scientists will lead which projects to maximize preferences?

(e) Do you support the assignment found in part (*d*)? Why or why not?

(f) Now you again consider all five scientists. You decide, however, that several scientists cannot lead certain projects. In particular, Dr. Mickey does not have experience with research on the immune system, so he cannot lead Project Hope. His family also has a history of manic-depression, and you feel that he would be too personally involved in Project Stable to serve as an effective project leader. Dr. Mickey therefore cannot lead Project Stable. Dr. Kvaal also does not have experience with research on the immune systems and cannot lead Project Hope. In addition, Dr. Kvaal cannot lead Project Release because he does not have experience with research on the cardiovascular system. Finally, Dr. Rollins cannot lead Project Up because her family has a history of depression and you feel she would be too personally involved in the project to serve as an effective leader. Because Dr. Mickey and Dr. Kvaal cannot lead two of the five projects, they each have only 600 bid points. Dr. Rollins has only 800 bid points because she cannot lead one of the five projects. The following table provides the new bids of Dr. Mickey, Dr. Kvaal, and Dr. Rollins:

Project	**Dr. Mickey**	**Dr. Kvaal**	**Dr. Rollins.**
Project Up	300	86	Can't lead
Project Stable	Can't lead	343	50
Project Choice	125	171	50
Project Hope	Can't lead	Can't lead	100
Project Release	175	Can't lead	600

Which scientists should lead which projects to maximize preferences?

(g) You decide that Project Hope and Project Release are too complex to be led by only one scientist. Therefore, each of these projects will be assigned two scientists as project leaders. You decide to hire two more scientists in order to staff all projects: Dr. Arriaga and Dr. Santos. Because of religious reasons, the two doctors both do not want to lead Project Choice. The following table lists all projects, scientists, and their bids.

	Kvaal	**Zuner**	**Tsai**	**Mickey**	**Rollins**	**Arriaga**	**Santos**
Up	86	0	100	300	Can't lead	250	111
Stable	343	200	100	Can't lead	50	250	1
Choice	171	800	100	125	50	Can't lead	Can't lead
Hope	Can't lead	0	100	Can't lead	100	250	333
Release	Can't lead	0	600	175	600	250	555

Which scientists should lead which projects to maximize preferences?

(h) Do you think it is wise to base your decision in part (*g*) only on an optimal solution for an assignment problem?

PREVIEWS OF ADDED CASES ON (www.mhhe.com/hillier)

CASE 8.2 Continuation of the Texago Case Study

The supplement to this chapter on the book's website presents a case study of how the Texago Corp. solved many transportation problems to help make its decision regarding where to locate its new oil refinery. Management now needs to address the question of whether the capacity of the new refinery should be made somewhat larger than originally planned. This will require formulating and solving some additional transportation problems. A key part of the analysis then will involve combining two transportation problems into a single linear programming model that simultaneously considers the shipping of crude oil from the oil fields to the refineries and the shipping of final product from the refineries to the distribution centers. A memo to management summarizing your results and recommendations also needs to be written.

CASE 8.3 Project Pickings

This case focuses on a series of applications of the assignment problem for a pharmaceutical manufacturing company. The decision has been made to undertake five research and development projects to attempt to develop new drugs that will treat five specific types of medical ailments. Five senior scientists are available to lead these projects as project directors. The problem now is to decide on how to assign these scientists to the projects on a one-to-one basis. A variety of likely scenarios need to be considered.

Solution CASES (TP and AP)

CASE 8.1 Shipping Wood to Market

Option 1:

		Unit Cost (1,000's) Destination (Market)					
		1	2	3	4	5	Supply
	1	61	72	45	55	66	15
Source 1	2	69	78	60	49	56	20
	3	59	66	63	61	47	15
Demand		11	12	9	10	8	

		Unit Cost (1,000's) Destination (Market)							
		1	2	3	4	5	Totals		Supply
	1	6	0	9	0	0	15	=	15
Source 1	2	2	0	0	10	8	20	=	20
	3	3	12	0	0	0	15	=	15
Totals		11	12	9	10	8			
		=	=	=	=	=	Total Cost	=	2,816.00
Demand		11	12	9	10	8			

Option 2:

		Unit Cost (1,000's) Destination (Market)					
		1	2	3	4	5	Supply
	1	58.5	68.3	47.8	55	63.5	15
Source 1	2	65.3	74.8	55	49	57.5	20
	3	59	61.3	63.5	58.8	50	15
Demand		11	12	9	10	8	

		Unit Cost (1,000's) Destination (Market)							
		1	2	3	4	5	Totals		Supply
	1	6	0	9	0	0	15	=	15
Source 1	2	5	0	0	10	5	20	=	20
	3	0	12	0	0	3	15	=	15
Totals		11	12	9	10	8			
		=	=	=	=	=	Total Cost	=	2,770.80
Demand		11	12	9	10	8			

Option 3:

		Unit Cost (1,000's) Destination (Market)					
		1	2	3	4	5	Supply
	1	58.5	68.3	45	55	63.5	15
Source 1	2	65.3	74.8	55	49	56	20
	3	59	61.3	63	58.8	47	15
Demand		11	12	9	10	8	

		Unit Cost (1,000's) Destination (Market)							
		1	2	3	4	5	Totals		Supply
	1	6	0	9	0	0	15	=	15
Source 1	2	5	0	0	10	5	20	=	20
	3	0	12	0	0	3	15	=	15
Totals		11	12	9	10	8			
		=	=	=	=	=	Total Cost	=	2,729.10
Demand		11	12	9	10	8			

The combination plan, i.e., shipping by either rail or water offers the best cost whereas shipping by rail is the most expensive. If the costs of shipping by water are expected to rise considerably more than those of shipping by rail, it is best to use option 1 and ship by rail. If the reverse is true, then it is better to use option 2. If the cost comparisons will remain roughly the same, then using option 3 is best. This option is clearly the most feasible, but it may not be chosen if it is logistically too cumbersome. Further information is needed to determine this.

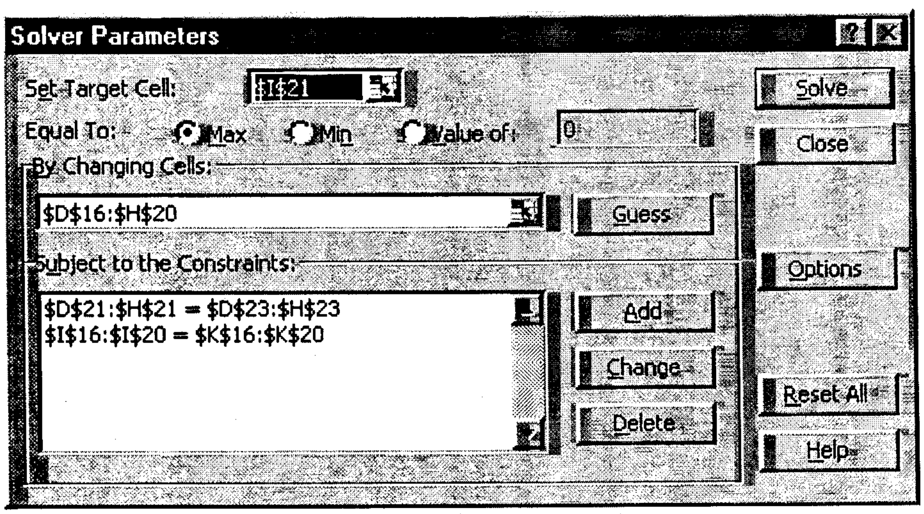
The problem in this case can be solved using assignment problem.

8.3 Throughout this case, we use the template for the assignment problem.

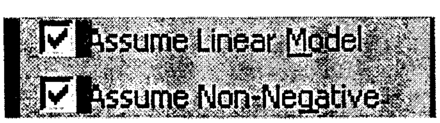
a) The projects are the tasks, and the scientists are the assignees in this assignment problem.

	A	B	C	D	E	F	G	H	I	J	K
1											
2				Points							
3				Task							
4				Up	Stable	Choice	Hope	Release	Supply		
5			Kvaal	100	400	200	200	100	1		
6		Assignee	Zuner	0	200	800	0	0	1		
7			Tsai	100	100	100	100	600	1		
8			Mickey	267	153	99	451	30	1		
9			Rollins	100	33	33	34	800	1		
10		Demand		1	1	1	1	1			
11											
12											
13				Assignments							
14				Task							
15				Up	Stable	Choice	Hope	Release	Totals		Supply
16			Kvaal	0	1	0	0	0	1	=	1
17		Assignee	Zuner	0	0	1	0	0	1	=	1
18			Tsai	1	0	0	0	0	1	=	1
19			Mickey	0	0	0	1	0	1	=	1
20			Rollins	0	0	0	0	1	1	=	1
21		Totals		1	1	1	1	1	2551	=	Total Points
22				=	=	=	=	=			
23		Demand		1	1	1	1	1			

The solver dialogue box appears as follows:



The solver options throughout this case are:



To maximize the scientists preferences you want to assign Dr. Tsai to lead project Up, Dr. Kvaal to lead project Stable, Dr. Zuner to lead project Choice, Dr. Mickey to lead project Hope, and Dr. Rollins to lead project Release.

b) Since there are only four assignees we introduce a dummy assignee with preferences of −1. The task that gets assigned the dummy assignee will not be done.

	A	B	C	D	E	F	G	H	I	J	K
1											
2						Points					
3						Task					
4				Up	Stable	Choice	Hope	Release	Supply		
5			Kvaal	100	400	200	200	100	1		
6		Assignee	Zuner	0	200	800	0	0	1		
7			Tsai	100	100	100	100	600	1		
8			Mickey	267	153	99	451	30	1		
9			dummy	-1	-1	-1	-1	-1	1		
10		Demand		1	1	1	1	1			
11											
12											
13						Assignments					
14						Task					
15				Up	Stable	Choice	Hope	Release	Totals		Supply
16			Kvaal	0	1	0	0	0	1	=	1
17		Assignee	Zuner	0	0	1	0	0	1	=	1
18			Tsai	0	0	0	0	1	1	=	1
19			Mickey	0	0	0	1	0	1	=	1
20			dummy	1	0	0	0	0	1	=	1
21		Totals		1	1	1	1	1	2250	=	Total Points
22				=	=	=	=	=			
23		Demand		1	1	1	1	1			

The solver dialogue box remains the same.

We give up on project Up.

c) Since two of the assignees can do two tasks we need to double them. We include assignees Zuner-1, Zuner-2, Mickey-1, and Mickey-2 into the problem. In order to have an equal number of assignees and tasks we also need to include one dummy task. In order to ensure that neither Dr. Kvaal nor Dr. Tsai can get assigned the dummy task and thus no project, we insert a large negative number as their point bid for the dummy project.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2				Points								
3				Task								
4				Up	Stable	Choice	Hope	Release	dummy	Supply		
5			Kvaal	100	400	200	200	100	-10000	1		
6			Zuner-1	0	200	800	0	0	-1	1		
7		Assignee	Zuner-2	0	200	800	0	0	-1	1		
8			Tsai	100	100	100	100	600	-10000	1		
9			Mickey-1	267	153	99	451	30	-1	1		
10			Mickey-2	267	153	99	451	30	-1	1		
11		Demand		1	1	1	1	1	1			
12												
13												
14				Assignments								
15				Task								
16				Up	Stable	Choice	Hope	Release	dummy	Totals		Supply
17			Kvaal	0	1	0	0	0	0	1	=	1
18			Zuner-1	0	0	0	0	0	1	1	=	1
19		Assignee	Zuner-2	0	0	1	0	0	0	1	=	1
20			Tsai	0	0	0	0	1	0	1	=	1
21			Mickey-1	0	0	0	1	0	0	1	=	1
22			Mickey-2	1	0	0	0	0	0	1	=	1
23		Totals		1	1	1	1	1	1	2517	=	Total Points
24				=	=	=	=	=	=			
25		Demand		1	1	1	1	1	1			

Dr. Kvaal leads project Stable, Dr. Zuner leads project Choice, Dr. Tsai leads project Release, and Dr. Mickey leads the projects Hope and Up.

d) Under the new bids of Dr. Zuner the assignment does not change:

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2				Points								
3				Task								
4				Up	Stable	Choice	Hope	Release	dummy	Supply		
5			Kvaal	100	400	200	200	100	-10000	1		
6			Zuner-1	20	450	451	39	40	-1	1		
7		Assignee	Zuner-2	20	450	451	39	40	-1	1		
8			Tsai	100	100	100	100	600	-10000	1		
9			Mickey-1	267	153	99	451	30	-1	1		
10			Mickey-2	267	153	99	451	30	-1	1		
11		Demand		1	1	1	1	1	1			
12												
13												
14				Assignments								
15				Task								
16				Up	Stable	Choice	Hope	Release	dummy	Totals		Supply
17			Kvaal	0	1	0	0	0	0	1	=	1
18			Zuner-1	0	0	1	0	0	0	1	=	1
19		Assignee	Zuner-2	0	0	0	0	0	1	1	=	1
20			Tsai	0	0	0	0	1	0	1	=	1
21			Mickey-1	0	0	0	1	0	0	1	=	1
22			Mickey-2	1	0	0	0	0	0	1	=	1
23		Totals		1	1	1	1	1	1	2468	=	Total Points
24				=	=	=	=	=	=			
25		Demand		1	1	1	1	1	1			

e) Certainly Dr. Zuner could be disappointed that she is not assigned to project Stable, especially when she expressed a higher preference for that project than the scientist assigned. The optimal solution maximizes the preferences overall, but individual scientists may be disappointed. We should therefore make sure to communicate the reasoning behind the assignments to the scientists.

f) Whenever a scientist cannot lead a particular project we use a large negative number as the point bid.

	A	B	C	D	E	F	G	H	I	J	K
1											
2						Points					
3						Task					
4				Up	Stable	Choice	Hope	Release	Supply		
5			Kvaal	86	343	171	-10000	-10000	1		
6		Assignee	Zuner	0	200	800	0	0	1		
7			Tsai	100	100	100	100	600	1		
8			Mickey	300	-10000	125	-10000	175	1		
9			Rollins	-10000	50	50	100	600	1		
10		Demand		1	1	1	1	1			
11											
12											
13						Assignments					
14						Task					
15				Up	Stable	Choice	Hope	Release	Totals		Supply
16			Kvaal	0	1	0	0	0	1	=	1
17		Assignee	Zuner	0	0	1	0	0	1	=	1
18			Tsai	0	0	0	1	0	1	=	1
19			Mickey	1	0	0	0	0	1	=	1
20			Rollins	0	0	0	0	1	1	=	1
21		Totals		1	1	1	1	1	2143	=	Total Points
22				=	=	=	=	=			
23		Demand		1	1	1	1	1			

Dr. Kvaal leads project Stable, Dr. Zuner leads project Choice, Dr. Tsai leads project Hope, Dr. Mickey leads project Up, and Dr. Rollins leads project Release.

g) When we want to assign two assignees to the same task we need to duplicate that task.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2							Points						
3							Task						
4				Up	Stable	Choice	Hope-A	Hope-B	Release-A	Release-B	Supply		
5			Kvaal	86	343	171	-10000	-10000	-10000	-10000	1		
6		Assignee	Zuner	0	200	800	0	0	0	0	1		
7			Tsai	100	100	100	100	100	600	600	1		
8			Mickey	300	-10000	125	-10000	-10000	175	175	1		
9			Rollins	-10000	50	50	100	100	600	600	1		
10			Arriaga	250	250	-10000	250	250	250	250	1		
11			Santos	111	1	-10000	333	333	555	555	1		
12		Demand		1	1	1	1	1	1	1			
13													
14													
15							Assignments						
16							Task						
17				Up	Stable	Choice	Hope-A	Hope-B	Release-A	Release-B	Totals		Supply
18			Kvaal	0	1	0	0	0	0	0	1	=	1
19		Assignee	Zuner	0	0	1	0	0	0	0	1	=	1
20			Tsai	0	0	0	0	0	1	0	1	=	1
21			Mickey	1	0	0	0	0	0	0	1	=	1
22			Rollins	0	0	0	0	0	0	1	1	=	1
23			Arriaga	0	0	0	0	1	0	0	1	=	1
24			Santos	0	0	0	1	0	0	0	1	=	1
25		Totals		1	1	1	1	1	1	1	3226	=	Total Points
26				=	=	=	=	=	=	=			
27		Demand		1	1	1	1	1	1	1			

Project Up is led by Dr. Mickey, Stable by Dr. Kvaal, Choice by Dr. Zuner, Hope by Dr. Arriaga and Dr. Santos, and Release by Dr. Tsai and Dr. Rollins.

h) No. Maximizing overall preferences does not maximize individual preferences. Scientists who do not get their first choice may become resentful and therefore lack the motivation to lead their assigned project. For example, in the optimal solution of part (g), Dr. Santos clearly elected project Release as his first choice, but he was assigned to lead project Hope.

In addition, maximizing preferences ignores other considerations that should be factored into the assignment decision. For example, the scientist with the highest preference for a project may not be the scientist most qualified to lead the project.